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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

Volume XXXVII Number 3

Edited by William David Reeve

Analytic Geometry: The Discovery of Fermat and Descartes

By Carl B. Boyer
Brooklyn College, Brooklyn, New York

Until a century ago the discovery of analytic geometry was ascribed categorically to the father of modern philosophy—René Descartes. *Proles sine matre creata,* Chasles called it, but this famous characterization is now quoted only to be refuted. The offspring is now recognized not only as having arrived as twins—for Pierre de Fermat had developed substantially the same method at about the same time—but also as having descended from a long line of legitimate antecedents. Recently attempts have been made to shift responsibility for the birth from the shoulders of Descartes and Fermat to those of one or more of the ancestors—Harriot or Cataldi or Ghetaldi or Viète or Benedetti or Oresme or Apollonius or the two thousand year old grandfather Menaechmus. In the case of any one of these individuals arguments may be advanced in his support. These can be based upon a definition of analytic geometry which is so framed as to include the work of the candidate in question and to exclude that of his rivals with respect to priority. The plausibility of the arguments is easily enhanced through an ingenious interpretation of the man's statements in terms of later symbolisms and concepts, thus implying a specious modernity of viewpoint. Few branches of mathematics lean as heavily upon a felicitous choice of notations as do those two powerful tools of science which we call analytic geometry and the calculus. It is in fact precisely the algorithmic nature of these subjects which represents the secret of their effectiveness. In view of this, great caution must be exercised in superimposing modern notations upon ancient treatises. Conventional symbolisms have been devised *ad hoc* to express contemporary concepts which have developed slowly and laboriously, and through repeated use such notations have come to be associated implicitly with the ideas which they are now intended to represent. Projection of such notations into antiquity leads easily to a corresponding anachronism with respect to concepts, as the history of analytic geometry clearly demonstrates.

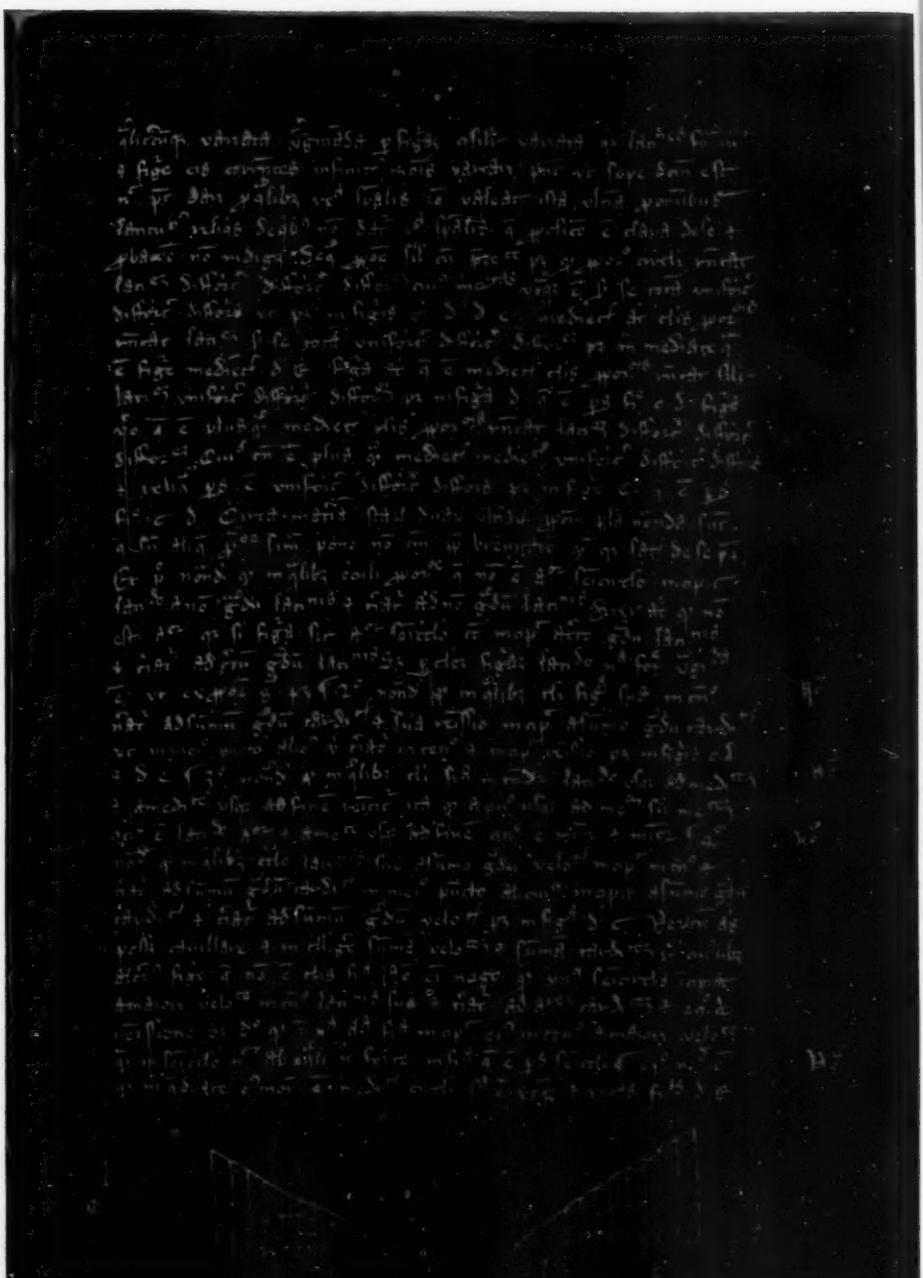
The query, "What constitutes analytic geometry?" has been variously answered. It is frequently characterized roughly as the combination of algebra and geometry. The utter inadequacy of this statement becomes apparent when one recalls that the ancient Pythagorean solution of quadratic equations through the application of areas would fall within this description and yet bears no significant resemblance to our analytic methods. Nor is one justified in holding that analytic geometry consists

simply of the introduction of analytic methods of reasoning into the subject matter of geometry. This would result in applying the name to work which is far removed from Cartesian geometry. Plato is commonly regarded as having suggested the use of the analytic type of argument—of proceeding from the conclusion to the premise—although in a broad sense it can be said to have been employed much earlier. Plato, however, had no vision of the part this was to play later in Cartesian geometry.

The use of coordinate systems often has been regarded as the feature distinguishing analytic methods from other approaches in geometry. That this is a necessary aspect of the subject can scarcely be denied, but in no sense is it to be regarded as a sufficient criterion. The ancient Egyptians apparently made use of schemes of rectangular coordinates in their cadastral surveys; and Hipparchus applied such methods more systematically, both to the geography of the Mediterranean region and in the construction of star charts. In the field of geometry coordinate methods were certainly used by Apollonius in his study of the properties of conic sections, if indeed these may not have been employed in this connection a century earlier by Menaechmus. Yet the point of view and the method of attack of Apollonius and Menaechmus were essentially different from those indicated by Fermat and Descartes. A closer examination of their work will reveal that the coordinate frame was in every case simply an auxiliary construction superimposed a posteriori on a given curve in order to *study* its properties. It was not used as a means a priori of *defining* the locus or of determining the generation of the curve. That is, Menaechmus, in attempting to solve the problem of the duplication of the cube by means of the continued proportion of Hippocrates (expressed in modern symbolism as $\frac{a}{x}=\frac{x}{y}=\frac{y}{b}$), had no conception of the relationships $ay=x^2$, $y^2=xb$ and $ab=xy$ as in themselves *defining* certain plane loci, but sought rather to *discover* curves otherwise defined, the points of which should possess these properties. This search led him, as is well known, to the sections of a cone. The very fact that conics were known in Greek geometry as "solid loci" indicates that they were defined stereometrically rather than analytically. More than 600 years later Pappus spoke likewise of "linear loci"—i.e., curves other than straight lines, circles, and conics—as "being generated from more irregular surfaces and intricate movements." They were not given by equations. Once a curve had been thus defined, it was of course not difficult to discover innumerable relationships in terms of lines associated with the curve. Such properties when symbolically expressed may in a sense be regarded as equations corresponding to the curves, and for this reason the origin of analytic geometry recently has been claimed for Greek antiquity. But according to this point of view the Babylonians possessed analytic geometry by 2000 B.C.! They were familiar with the relationship $4r^2=c^2+4(r-s)^2$, where s is the sagitta of any chord of length c in a circle of radius r. This relationship *may* be regarded—although there is no evidence that the Babylonians did so—as the equation defining the circle in terms of the rectangular coordinates c and s. Thus interpreted, the Babylonians would be regarded as having used analytic geometry. Similarly the Euclidean propositions on the straight line and the circle can be looked upon as equations of these curves, and the *Elements* would then be an example of analytic geometry. One will protest that Euclid did not make use of the modern algebraic symbolism which is used in equations. This is true, but then neither did Menaechmus before him nor Apollonius somewhat later. Moreover, they are alike in that invariably *the equations are derived from the curve*, and not conversely. It is true that Apollonius and other Greek geometers used the stere-

ometric origin of the conics only so far as was necessary to deduce a single fundamental plane property (equation) for each curve, and that this latter result was then made basic in establishing further properties, such as those of the asymptotes. Nevertheless, it is significant that they felt it necessary to give the curves a geometrical definition distinct from that of points the coordinates of which satisfy a given equation. In fact, by "finding" a conic for which a given relationship held (i.e. satisfying a given equation) they understood not plotting it by points but by determining it as a section of a cone. While one may safely assert that in a broad sense Menaechmus and Apollonius made use of analytic methods, it is nevertheless not correct to say that they understood the fundamental principle of analytic geometry—that, in general, an equation in two variables defines a plane curve in a given coordinate system such that all pairs of values satisfying the equation are coordinates of points on a curve; and, conversely, all points on this curve have coordinates which satisfy the given equation. In part the failure of Menaechmus and Apollonius to appreciate this principle may have resulted from the weakness of contemporary algebra. It may also have been the consequence of a peculiar deficiency in Greek thought—the lack in mathematics of the notion of a variable. The paradoxes of Zeno had left a profound impression which caused the generic ideas of change and variability to be relegated to metaphysics. Geometric magnitudes were static and continuous; algebraic quantities were discrete constants. The symbols in the *Arithmetic* of Diophantus represent unknown numbers rather than variables in the sense of elementary algebra. Modern higher analysis may indeed be founded logically upon the Weierstrassian "static theory of the variable" which represents in a sense of partial return to the ancient Greek view; but the considerations leading to the original development of analytic geometry (as also of the calculus) were definitely phoronomic. During the later Scholastic period, and particularly during the fourteenth century, there arose at Paris and Oxford lively discussions on questions concerning rates of change, both uniform and non-uniform. The ideas of inertia and of acceleration, ubiquitously but erroneously ascribed today to Galileo, were developed at this time. In connection with this work Nicole Oresme made one of the earliest attempts to represent graphically the manner in which one quantity varied with another. For example, the velocity of a freely falling body, which was taken to be proportional to the time of fall, he plotted as a function of the time. Marking off points on a horizontal straight line as time units, the velocity corresponding to any time was represented as an ordinate at this point, and the resulting graph was an oblique straight line. Oresme attempted to picture also certain functions which were "difformly difform" (that is, the rate of change was not constant), the graphs in these cases being necessarily curvilinear.

In certain quarters Oresme has been vigorously acclaimed as the inventor of analytic geometry. His work undoubtedly represents a very definite step in this direction, for he may be looked upon as the father of systematic graphical representation, but his view is seen to fall short of the conceptions of Descartes and Fermat. In a sense one may say that Oresme showed that a simple proportion can be represented by a straight line and that certain other types of variation are associated with characteristic diagrams. This may be regarded as an advance beyond the coordinate considerations of Apollonius in that the curve is here determined by the coordinate system and by the law of variation; the coordinate frame is not introduced as a device for studying a curve already given. Nevertheless, Oresme was prevented from taking full advantage of his novel idea by deficiencies in geometrical knowledge and algebraic technique. Unfortunately the mathematical interest of the age took the form of abortive spec-



GRAPHS OF ABOUT 1361

Photostat of a page (fol. 205^{r}) of a manuscript copy (Bayerische Staatsbibliothek, München, cod. lat. 26889, fol. 201^{r}–206^{r}) of the *Tractatus de latitudinibus formarum*. This work usually is ascribed to Nicole Oresme, but it is probably a student's notes on Oresme's longer treatise, *Tractatus de figuratione potentiarum et mensurarum*. For accounts of these see references 6, 25, and 27. For another facsimile of a page from the *Latitude of Forms* see page 276 of reference 9.

ulations (resumed half a millenium later by Cantor) on infinity and the continuum, and the Greek classics were neglected. Consequently there is in the work of Oresme no systematic association of algebra and geometry in which an *equation* in two variables determines a specific curve and conversely.

The shortcomings in algebra which are apparent in the work of Menaechmus, Apollonius, and Oresme were not removed until the sixteenth century. The arithmetic of Diophantus (and, a fortiori, that of the Babylonians) had remained largely unknown to the Latin medieval world, but by the Arabs it had been translated, amplified, and transmitted to Europe. Although Leonardo of Pisa appreciated the significance of this work, it remained latent in Europe for almost three centuries before algebra was systematically developed by Chuquet, Pacioli, Cardan, Stifel, and many others. This development happily coincided with a strong revival of interest in ancient geometry, including the *Conics* of Apollonius. Not unnaturally the gap between geometric and algebraic methods became less pronounced than it had been in the works of the classical Greek period. Tartaglia, Cardan, and Bombelli solved equations geometrically and, inversely, algebraic methods were widely advocated by Benedetti, Clavius, Viète, Stevin, Girard, Ghetaldi, Cataldi, Harriot, and Oughtred as a means of simplifying geometrical problems. Viète realized as well the advantage to be gained by Plato's analytic approach. Moreover, he contributed to algebra a significant point of view: the subject was to be looked upon not simply as numerical logistic but as the logic of magnitudes in general—*logistica speciosa*. These magnitudes he represented by letters, vowels for the unknown and consonants for those assumed known. It is to be remarked clearly, however, that these symbols denoted determinate quantities rather than variables. The geometric problems were reducible to equations in a single unknown, and the value of this unknown was determined by solving this equation. In fact, the algebra of the time was predominantly concerned with the solution of such equations. Diophantus had considered equations in several unknowns from the point of view of the theory of numbers; but these did not attract the algebraist of the sixteenth century, for they could not be "solved" in the ordinary sense. When a geometric problem led to an equation in two unknowns, it was abandoned as unsolvable. This is where the genius of Fermat and Descartes appeared. These men saw that lack of a unique solution did not in this case render the problem devoid of interest. They interpreted the one unknown as a variable horizontal line segment (abscissa) with a fixed initial end-point (origin), and at the other extremities of this they erected perpendicular line segments (ordinates) of lengths corresponding to the values determined for the second unknown in terms of the first. The extremities of these ordinates, infinite in number, formed a curve which, for a given equation and coordinate system, was uniquely determined. As Fermat stated it (*Oeuvres*, I, 91; III, 85): "whenever in a final equation two unknown quantities are found, we have a locus, the extremity of one of these [unknown magnitudes] describing a line, straight or curved."

That the crux of the matter lies in the interpretation of problems leading to a final equation in more than one unknown is quite clear also from the work of Descartes: Book I deals with determinate problems; i.e., those which lead to "as many such equations as there are supposed to be unknown lines." (*Geometry*, p. 6.) Book II, on the other hand, deals with plane problems on loci in which "this [final] condition can be expressed by a single equation in two unknown quantities." (*Geometry*, p. 34.) In suggesting the extension of this discovery to three dimensions he adds: "If two conditions for the determination of the point are lacking [i.e., if the final equation has three un-

knowns], the locus of the point is a surface." (*Geometry*, p. 80.) Fermat stated the principle still more sharply as follows: "There are certain problems which involve only one unknown, and which can be called determinate, to distinguish them from the problems of loci. There are certain others which involve two unknowns and which can never be reduced to a single one: these are the problems of loci. In the first problems we seek a unique point, in the latter a curve. But if the proposed problem involves three unknowns, one has to find, to satisfy the question, not only a point or a curve, but an entire surface." (*Oeuvres*, III, 161.)

The object of this paper has not been to decide whether or not Descartes and Fermat invented analytic geometry. An answer to this problem depends on one's definition and sense of proportion; but *de gustibus non est disputandem*. The aim has been the simpler one of indicating just what it was that these men did discover in this connection. Neither one of them invented the use of coordinates or of the analytic method. Neither was first in applying algebra to geometry or in graphically representing variables. Moreover, it had long been known that, for a given curve, certain distance relationships are determined which may be interpreted as equations of the curve with respect to coordinate systems. However, there appears to have been no conception before their time of the converse—the fact that in general an arbitrary *given equation* involving *two unknown quantities* can be regarded as determining *per se*, with respect to a coordinate system, a plane curve. This latter recognition, together with its fabrication into a formalized algorithmic procedure, constituted the decisive contribution of Fermat and Descartes. This is the sense in which these men may be regarded as the founders of analytic geometry, much as Newton and Leibnitz are generally regarded as the inventors of the calculus through precise formulation of the mutually inverse nature of area and tangent problems in terms of a definitely regularized operational procedure. The publication of Descartes' *Géométrie* in 1637 preceded by 42 years that of Fermat's *Isagoge*, but both men were in independent possession of their methods well before this time—about 1619 for Descartes and 1629 for Fermat. As in the calculus the root idea was due to the two greatest figures of the second half of the seventeenth century, so likewise in analytic geometry the fundamental principle was recognized by the two greatest mathematicians of the first half of this same century.

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A further contribution by Gino Loria, "Perfectionnements, évolution, métamorphoses du concepte de coordonnées. Contribution à l'histoire de la géométrie analytique," was scheduled for publication in volume VIII of *Osiris*, but the appearance of this volume has been held up by the German invasion of Belgium.

How adequate schools, adequately equipped, and better paid teachers bring a dollar and cents return to community, state and nation, is the theme of a new sound motion picture, "Pop Rings The Bell." Production of this motion picture has been assigned to The Jam Handy Organization, Detroit, Mich., by its sponsors—The National School Service Institute.

Upon completion, prints or copies in 16 mm. will be made available for showings before special groups. The picture dramatizes the new and growing needs of schools to provide the kind of education the community and Nation must have to meet the new demands inevitable in the postwar world.

A Report of the Radio Committee of the National Council of Teachers of Mathematics[1]

By A. Brown Miller
West Technical High School, Cleveland, Ohio

A little more than two years ago a committee of the National Council of Teachers of Mathematics began a survey of the use of the radio and public address systems in our public schools. One hundred public school systems with populations of 50,000 or more throughout the United States were sent questionnaires. Significant replies were received from those listed in the report. Other school systems replied and indicated an interest in the results of the questionnaire. After receiving the first replies they were returned to each school system in September 1942 to be rechecked. The report as now given therefore represents the situation as of November 1942.

It is hoped that this initial report will be of assistance to school systems in finding avenues of cooperation with others who are working along similar lines. For our committee it furnishes the foundation upon which we hope to build a program of research regarding the use of radio in the field of mathematics. The correspondence indicates a keen interest in this report and that splendid pioneering work is being done. Our problem is to make available to all the experience of each through an interchange of ideas.

Our report surveyed the complete area of the use of the radio in order to determine the facilities now available and installed. These can be used in any field as materials become available. It is evident that teachers of mathematics are making all too little use of the facilities available. The immediate demand is for scripts, plays, biographical recordings, historical recordings, model lessons, etc. Wherever public address systems are installed these could be used. The report indicates the school systems which have made an attempt along these lines. We are giving the names of persons in these school systems who are directly responsible for these beginnings or are interested in the creation of materials.

The Cleveland Ohio Schools are the pioneers in an attempt to use the radio as a means of instruction in the field of mathematics. One lesson a week is now being given in arithmetic in all of the elementary schools. This work has been in progress for a number of years and is beyond the experimental period and in rather definite form.

This report does not include the contribution to public schools by broadcasts from colleges and universities. In certain fields of instruction these are significant.

The committee has accomplished its purpose if it has succeeded in indicating:

1. the scope of the use of the radio in the field of public education
2. the extent to which the facilities are being used in the field of mathematics
3. the school systems where work of the kind that you are interested in doing is being attempted.

Now we invite correspondence, inquiries, and information that will assist us in aiding all who are interested in our common problems of making significant use of the radio and the public address system in the field of mathematics.

The following persons are named in the

[1] A number of the cities included in the above report have in the past year greatly improved their work in radio and the committee hopes to reevaluate the report by getting a more detailed account of the present situation in those cities soon. We would appreciate knowing particularly of the work in any system where classroom instruction is given by radio.

reports as being particularly interested or at work on committees directing radio activity:

Baltimore	Md.	Nannette L Roche	Supervisor of Mathematics
Birmingham	Ala.	Ralph Martin	General Radio Committee
Boston	Mass.	Joseph A Hennessay	Teachers College Radio Committee
Brooklyn	N. Y.	George Ross	H. S. Automotive Trades, 50 Bedford Avenue
Chicago	Ill.	George Jennings	Acting Director, Radio Council
Cleveland	Ohio	Dr. H. E. Grime	Supervisor of Mathematics
Detroit	Mich.	W. W. Whittinghill	Dept. of Visual and Radio Education, 9345 Lawton Ave.
Flint	Mich.	Lowell E. Grant	Radio Education Committee
Fort Worth	Texas	W. E. King	Consultant of Instruction Science and Mathematics
Gary	Ind.	R. G. Anderson	
Huntington	W. Va.	Dorothy B. Bragonier	
Indianapolis	Ind.	Robert T. Harrison	Director of Radio Activities
Long Beach	Calif.	W. J. Klopp	Supervisor of Sec. Ed.
Milwaukee	Wis	Hasel Hesley	Washington H. S.
Nashville	Tenn.	L. G. Deitrich	Assistant Supt.
Pontiac	Mich.	Ola Hiller	Charge of Radio
Richmond	Va.	Mrs. F. M. Lohman	Chairman Radio Committee
Portland	Ore.	Lesta Hoel	Supervisor of Mathematics
Salt Lake City	Utah	Ernest M. Hanson	Supervisor of Mathematics
San Diego	Calif.	Mary Maull	Coordinator of Instructional Aids
Spokane	Wash.	Kate Bell	Lewis & Clark H. S.
St. Louis	Mo.	C. R. Aydelott	Director of instruction and curriculum
Toledo	Ohio	Judson C. Siddell	Board of Education
Washington	D. C.	Veryl Schult	Head of Mathematics Div. 1–9

This questionnaire was sent to the school systems from which this report is compiled.

Questionnaire

DEAR SIR:—

A committee of the National Council of Teachers of Mathematics is making a study of the use of the radio in the field of mathematics. A preliminary investigation shows that some Boards of Education and educational institutions own and operate their own stations, others use time of local privately owned stations, and others are limited to public address systems within buildings. Cleveland, Ohio, e.g., owns and operates its own station with a full schedule each day devoted to various fields. Mathematics takes its place in this schedule broadcasting a lesson once a week to the first four grades of elementary arithmetic. In many places splendid pioneering work is being done which would be helpful to others if known. This committee hopes by this questionnaire to learn of these and make them known through the Council's monthly publication, THE MATHEMATICS TEACHER, now read by more than five thousand subscribers throughout the nation. Will you please cooperate with us by having this questionnaire answered and returned promptly?

1. Does your Board of Education or Institution own and operate a radio station? _____
   (a) How much time per day or week is it operated? _____
   (b) What subjects are taught or supplemented by use of the radio? _____
   _____
   (c) Do you have any reports descriptive of radio activities which you could include for our use for which we are willing to pay? _____
   _____
2. Do you use time on locally owned stations? _____
   (a) How much time is used? _____
   (b) What subjects are emphasized?
   _____
   (c) If mathematics is a part of this program would you please include in this reply a brief description of what is done. We will be glad to pay for any printed descriptive material.
3. How many of your buildings are equipped with public address systems through which subject matter material is now, or could be, given to pupils in their classrooms?
   (a) In what way does your mathematics department use this system? _____
   _____
   (b) Have you produced any plays or recordings for use over the system?
   _____
   (c) Please enclose a description of all the activities in the field of mathematics which you are using.
4. Is there a committee or individual in your school system studying this problem of the use of the radio or public address system in the field of mathematics? If so would you please include the name or names so that we may co-operate with them in this matter.
5. Please include in this answer any compiled reports or outlines which you think would be helpful to us in getting a complete picture of this situation. I know now that some schools have prepared short talks suitable for the pupils and general public on mathematics, New York City is outstanding in this field. Others have developed synchronized recordings and pictures dealing with certain topics in the social phases of mathematics and historical topics. We are interested in determining just what is being done in this field and where the Council may be helpful to the individual school.
6. Would you be interested in a report of this committee based upon the findings of this questionnaire?

Thanking you for your cooperation in this undertaking, I am,

Yours truly,

A. BROWN MILLER
Chairman of Radio Committee
of the National Council of
Teachers of Mathematics.

H. E. GRIME, Supervisor of Mathematics
Cleveland, Ohio
BEULAH SHOESMITH, Chicago, Ill.
ROBERT YEATES, Baton Rouge, La.
THEO DONNELLY, Milwaukee, Wis.

The table which follows on page 109 is a summary of the reports which were received from the cities indicated.

Use of Radio in Public Education		Own Station	Use Local Station	Hours Per Week	Public Address Systems	General Programs	Art	Dramatics	English	Health	Language	Mathematics	Music	Safety	Science	Social Studies	Public Relations	Mathematics Scripts	Mathematics Recordings	P A System for Math.	Scripts	Recordings	Plays	Materials Available
Albany	N.Y.		√		2																			
Altanta	Ga.		√	3																			√	
Baltimore	Md.		√	.5	2		√	√	√			√				√				√			√	√
Berkeley	Calif.		√	3	8	√		√	√				√			√	√				√			
Birmingham	Ala.		√	.5	2				√	√					√						√		√	
Boston	Mass.		√	1.5	√					√				√			√				√			
Brooklyn	N.Y.	WYNE		3	59				√		√	√				√				√	√		√	√
Charleston	S.C.		√		1	√															√			
Chicago	Ill.	WBEZ	√	5		√	√	√	√		√			√	√	√	√				√		√	√
Cincinnati	Ohio		√	1.25	6							√				√								
Cleveland	Ohio	WBOE		40	30	√	√	√	√	√	√	√	√	√	√	√	√	√		√	√	√	√	√
Denver	Col.		√	2	1																√			
Des Moines	Iowa		√		6	√															√			
Detroit	Mich.		√	2	55	√	√	√	√	√	√	√	√	√	√	√	√				√		√	√
Duluth	Minn.		√	.25																				
Flint	Mich.	SW	√	.25	√	√		√	√					√	√	√	√				√		√	√
Fort Wayne	Ind.		√		20	√	√	√	√					√	√	√	√	√			√		√	√
Fort Worth	Texas		√		4	√				√		√	√	√		√	√							
Gary	Ind.		√		√			√	√				√			√	√				√			
Grand Rapids	Mich.		√	.25		√					√		√	√	√	√	√				√			
Houston	Texas		√	1	1			√	√				√	√	√	√	√				√		√	√
Hartford	Conn.		√	.25	2	√			√							√								
Indianapolis	Ind.		√	3.25	6	√	√			√			√	√		√	√				√		√	
Kansas City	Kans.		√	.25	6	√																		
Kansas City	Mo.		√	.5	2	√		√													√		√	
Lincoln	Neb.		√	.5	1	√			√		√		√	√		√	√							
Long Beach	Cal.	W6LSM	√		2		√	√	√				√	√		√	√				√		√	
Louisville	Ky.		√	.5		√																		
Milwaukee	Wis.		√	2.5	1	√																		
Minneapolis	Minn.		√	3	8	√		√	√				√	√		√	√				√		√	√
Montgomery	Ala.		√		2																			
Nashville	Tenn.		√	1	8								√	√								√		
New Haven	Conn.		√	.5		√		√	√				√	√			√				√		√	√
New Orleans	La.		√	1	√				√							√	√							
New York	N.Y.	WNYC		3									√					√						√
Pittsburgh	Pa.		√	2.75	2	√				√			√			√	√							
Pontiac	Mich.		√	.5	1																			
Portland	Ore.	IBPS	√	20	√	√	√	√	√	√	√	√	√	√	√	√	√	√		√	√	√	√	√
Richmond	Va.		√	.25	4											√								
Rockford	Ill.		√		2	√							√											
Rochester	N.Y.		√	2.5	2	√							√		√		√							√
Salt Lake City	Utah		√	.5	6								√			√		√				√		
San Diego	Calif.		√	.5									√									√		
Spokane	Wash.		√	1.45	13	√	√	√	√	√	√	√	√	√	√	√	√	√				√	√	√
St. Louis	Mo.		√	.5	3	√																		
Toledo	Ohio		√	2.5	6	√		√	√	√	√		√	√	√	√	√				√		√	√
Tulsa	Okla.		√	1.5		√		√		√			√	√			√						√	√
Washington	D.C.		√	2.25	6	√	√	√	√	√			√	√	√	√	√					√		√

Demonstration Teaching in Arithmetic by Radio

By MARCELLA LEAHY MC NERNEY
Cleveland Public Schools, Cleveland, Ohio

THERE are many supervisory aids through which the in-service training of teachers may be attempted. However it is the purpose of this paper to discuss only one, radio as a tool in the hands of the supervisor. It is not enough to formulate a philosophy which guides the construction of a course-of-study or to issue a set of objectives which harmonizes with modern life. Most teachers need help in interpreting the course-of-study and in putting it into practice. Not all teachers are number conscious. They must develop alertness in recognizing and skill in effectively utilizing the number situations which arise in the classroom. They must develop the ability to plan other activities which require the pupils to do quantitative thinking or which provide the necessary drill for mastery of the skills. The effectiveness of a teacher's work is conditioned, in no small degree, by the quality of supervision.

The Cleveland experiment in radio has evolved about a definite philosophy which has, as its core, the belief that this new form of communication can facilitate the distribution of "expertness." Charles H. Lake, in the foreword to "Report of Radio Activities Station WBOE" states:

> We are convinced that many phases of school work can be improved by a reasonable amount of direct and indirect classroom instruction over the radio.
>
> We are convinced that while radio is not a substitute for the teacher, it is a device which extends to a remarkable degree the work of the good teacher and the influence of the supervisor and administrator.

GENERAL PROBLEMS

In planning a series of broadcasts to vitalize the arithmetic program and serve as demonstration lessons for teachers in the field, many problems had to be considered. The most important of these are listed below:

1. *X*, *Y*, *Z* ability grouping was followed in Cleveland.
2. There was a great heterogeneity of background of the pupils.
3. Promotion policies were largely on a chronological age basis.
4. Reading was by far the main factor in the primary grades.
5. There was no special supervisor of arithmetic until the year radio arithmetic was started.
6. The curriculum of the elementary school was already a crowded one.
7. There was a definite trend toward integration among many contemporary educators.
8. The teaching personnel consisted of specialists as well as non-specialists.
9. The absence of the personality of the teacher had to be taken into consideration.
10. There was a lack of printed material to guide one who planned to write radio lessons.

THE PROGRAM

In the Cleveland Public Schools all classification by grade levels has been eliminated in the primary division and has been replaced by levels of achievement. The primary course of study in arithmetic is divided into nine levels. In the kindergarten and early primary levels provision is made for regular, although not formal, number work. There may be no definite period set apart each day for instruction in arithmetic. The work is rather a part of real experiences in work and play. To the child, arithmetic should appear incidental to the things he wants to do, but to the teacher, it should be a part of a definitely organized plan to develop an interest in, and an understanding of, the use of number which forms the foundation for the work of later levels. Children

are not required to memorize any number facts until Level IV, which is generally reached by the average child in the third semester of school beyond kindergarten. Although no formal drill is used to fix facts until that level has been reached, many facts are actually learned in preceding levels because of their repetition in activity and play. In the succeeding levels groups of facts are drilled upon, and, the processes of addition, subtraction, and multiplication are introduced and developed systematically. However, throughout the entire nine levels much emphasis is placed upon meanings and insights. The course of study, for the upper elementary division is divided into twelve levels of attainment. Here again much stress is placed upon developing concepts and an understanding of the division process, fractions, etc., before formal drill and mastery are required.

The series of radio lessons under discussion covered Levels V to IX of the Primary Course of Study and Levels I and II of the Upper Elementary. The first broadcast was put on in October, 1938, and these lessons were continued for four successive semesters at the various levels as shown below. Primary Level V is generally reached by the average child in the fourth semester of school beyond kindergarten. The levels covered in the radio series, the number of lessons in each level, and the corresponding semester during which the normal child usually reaches each level follow:

Level	No. of Lessons	Semester in School Beyond Kindergarten	Date
Primary Level V	14	Fourth	Fall 1938
Primary Level VI	8	Fifth	Spring 1939
Primary Level VII	10		
Primary Level VIII	10	Sixth	Fall 1939
Primary Level IX	6		
Upper Elementary Level I	8	Seventh (Grade 4B)	Spring 1940
Upper Elementary Level II	7		

One lesson of approximately fifteen minutes in length was broadcast each week. A total of sixty-three lessons was given over the air during the four semesters of broadcasting. Approximately 125 teachers and 4,500 pupils received the lessons each semester.

Special Problems of the Radio Lesson

The special problems of the radio lesson are discussed under six headings: (1) Content and organization, (2) Script writing, (3) The teacher's guide, (4) The work sheet, (5) The Try-out, (6) Broadcasting.

Content and Organization

The Cleveland course of study had been revised just prior to the initial steps in the preparation of the radio series under discussion. Child interests and needs, adult needs, best practices in other cities and scientific research studies, when available, were among the criteria used in setting up the objectives. Although the course of study was in tentative form, it was assumed that all the material listed for a given level was worthwhile and should be taught. However, as only one lesson was to be given each week, not all of these objectives could be taught over the air. It was one of the problems of the writer to select and organize the material for the series as a whole and for each individual broadcast.

The first step in this process was to study the objectives and choose the most important ones, at the same time making sure that there was enough variety of topics to present a balanced program for the series. Then a survey was made of the

most recent and reputable textbooks of the achievement level for which the lessons were to be broadcast. Although it was not hoped to find radio lessons in these books, it was considered advisable as a matter of background to be familiar with the procedures and methods of the leading writers of arithmetic textbooks of the day.

Script Writing

In Cleveland the manuscripts for the radio lessons are prepared by either the special supervisor in a particular subject matter field or a teacher who has been freed from classroom duties during the time required to produce a lesson or series of lessons. The arithmetic lessons were prepared by the author of this paper, a classroom teacher at Milford School, the arithmetic curriculum center. There are twelve curriculum centers in Cleveland; each serves as a laboratory for the development of superior techniques in the teaching of a specific subject. The average length of time required to complete a lesson was approximately 25 hours. Because of the necessity of experimenting with various techniques and methods to find those best adapted to radio teaching, the preparation of the first lessons was naturally more time-consuming than the later ones.

Regardless of the success teachers have had in the classroom, radio teaching presents new difficulties. "Script writers must be oriented in the land of radio—of teaching the unseen—of popularizing, simplifying, and condensing."[1] Every step must be carefully planned and written into the manuscript. Every pupil reaction must be visualized and taken into consideration in planning succeeding parts of the lesson. A poorly stated sentence, an ambiguous direction, or a mistimed pause may throw the whole lesson off. There is no opportunity to see what is happening in the various classrooms and correct one's errors.

Standards used to guide in the writing of the arithmetic lessons might be summarized as follows:

1) Introduction should be interesting but brief. As only about fifteen minutes is devoted to the whole lesson, a relatively short time should be spent in "getting into" the lesson proper.
2) The vocabulary should be suited to the maturity and background of the listeners.
3) Provision should be made in the radio lessons for the defelopment of new words needed by the pupils to understand the processes and their application to problem stiuations.
4) Sentences should be neither short and choppy nor long and involved.
5) It should always be remembered that simple English is the most effective. Although a prepared manuscript is read before the microphone, it is the hope of the broadcaster that he will lead his audience to believe that they are listening to someone talking to them, not reading to them.
6) Before the lesson is finally put on the air, it should be fine-combed for needless repetition and any unnecessary statements.
7) The introduction of too many concepts in one lesson should be avoided.
8) The body of the lesson should be clear, interesting, and orderly in arrangement.
9) The ending of the lesson should be effective. The writer usually closes with a short summary of some type.
10) The material presented in the lesson should be carefully checked for accuracy and validity.

In order to approximate as nearly as possible real demonstration lessons the radio broadcasts were planned to incorporate classroom teacher activity as well as a great deal of pupil activity. A study of the scripts for the arithmetic lessons reveals a lengthy list of types of pupil activities. Some selections from this list are:

Answering questions
Discussing quantitative aspects of various problems
Finding days and dates on the calendar
Counting children, coins, and other articles
Filling in elliptical sentences
Planning various types of parties
Using measuring tools, such as ruler, yardstick, scales, cup, pint and quart measures
Writing on the blackboard and on individual work sheets

[1] Darrow, B. H., *Radio, the Assistant Teacher* Columbus, Ohio. R. G. Adams and Company, 1932, p. 271.

Telling time, making clock faces, moving hands on clock faces
Keeping game scores
Taking part in question bees
Making change
Taking care of money and blanks for seed order activity
Analyzing concrete problems
Reading temperatures on the thermometer
Making graphs
Dramatizing number facts
Estimating size and weight
Reading radio time schedules
Taking tests

Pauses ranging from 3 seconds to 120 seconds were allowed during which pupils or teacher performed the activity called for by the broadcaster. The average number of pauses in the lessons for Level V were sixty-five; the average length of these pauses 9 seconds. The length of each pause is indicated in the manuscript as shown in the following excerpt:

A girl may go to the blank calendar (*8 sec.*). Show your teacher where to put the name October (*5 sec.*). Will the teacher please print October (*10 sec.*)? Point to the two spaces for the year (*5 sec.*). Tell your teacher what to write in those spaces (*10 sec.*). Can you find the spaces for the names of the days (*5 sec.*)? Say the names of the days slowly and your teacher will print the abbreviations for them in the right boxes (*45 sec.*). You may sit down (*5 sec.*).

So that the eye may aid the ear in the reception of these lessons, visual aids have been brought in wherever it is feasible. Some of the materials used are listed below:

Calendars
Blackboard
Colored chalk, crayons, and pencils
Measuring tools and utensils
Coins and currency
Charts
Clocks
Work sheets, many illustrated
Games
Scales
Seed Order Blanks
Flash Cards
Radio Schedules
Record Cards

Aside from the values derived from visual aids during the broadcast are the benefits received from the use of this material in the succeeding lessons taught by the classroom teacher. If the material is there and the teacher has been instructed in the effective use of such, the chances are that it will be used again and again.

The Teacher's Guide

Good supervision is a cooperative enterprise. It is of paramount importance, therefore, that there be real team-work between the radio teacher and the classroom teacher. If the latter views the radio lessons as a real help and looks forward to them with pleasurable anticipation, the children will be apt to reflect her attitude and derive a greater profit from them. So that the classroom teacher may know beforehand the material presented in the radio lesson, printed guides to supplement each lesson are sent out in advance of the broadcast. It is most helpful to the classroom teacher to receive these guides at the beginning of the semester so that she may properly prepare her class for each broadcast and so that she may delay presenting the objectives which are to be initiated in the radio lessons.

The teacher's guides were organized to furnish the following information:

Before the Broadcast. The teacher's preparation of the pupils and materials for the radio lesson.

During the Broadcast. The teacher's responsibility during the lesson.
Types of teacher and pupil activity to be called for in the lesson.

Follow-Up. (1) After the Broadcast; (2) During the Week
Suggested activities.
Optional tasks for bright classes or bright individuals.
Suggestions for further use of work sheets for grouping of children, or for diagnostic purposes.
Samples work sheets to be used during the week.

References. Practice pages in the basic textbook and in supplementary texts on objectives presented in the radio lesson or to be presented by the classroom teacher during the week.

Objectives. List of objectives presented during the radio lesson and to be presented by the teacher during the week.

Vocabulary. List of words introduced which are of a quantitative or qualitative nature or which are used in process or problem work.

There was an earnest effort on the part of the writer to make the preparation of material before the broadcast as little time consuming as possible so that the classroom teacher would look upon the lessons not as an added burden but rather as an aid to her teaching. In the majority of cases this preparation could be made in less than fifteen minutes, while in many cases it consisted of merely having on each child's desk a pencil and a copy of the work sheet for the lesson. In cases where charts or games had to be made or materials had to be brought in, suggestions were made in the Follow-Up for a variety of ways in which such material could be used in other lessons. Lessons which required considerably more than the usual amount of preparation were succeeded by lessons which required relatively short periods for preparation.

The activities listed under the Follow-Up were usually suggestive, so that the classroom teacher could omit them if they did not fit the needs of her group of children. Many sample work sheets covering material not found in the textbooks were prepared and sent with the teacher's guide. An hour or two spent by the script writer could save that much time for each of the teachers taking the broadcasts.

The Work Sheet

Individual pupil work sheets designed to fit the script were used in the majority of the radio lessons. These sheets were planned by the script writer, mimeographed at the Board of Education Headquarters, and sent to each teacher a few days before the broadcast in which they were to be used. A copy of the pupil's work sheet was attached to the teacher's guide for each lesson and these guides were sent out in advance of the broadcast dates.

Many of the work sheets were illustrated to develop concepts of a process or to make vital and interesting, situations concerning which children were asked to solve problems. Other sheets contained problems about activities that were very apt to or at least might possibly arise in the Cleveland Public Schools. For instance during the Fall Semester bulb order blanks for individual pupils to take home are sent to every school in the system. In the Spring Semester similar blanks for bulbs and seeds are sent out. A work sheet was designed to show how to tabulate the data for such an activity when an individual class takes charge of the orders and money for the entire school or a group of rooms, such as the primary section, the upper elementary section, the first floor rooms, or the second floor rooms. Another work sheet was planned to present problems taken from the individual Height and Weight cards kept for every child in the system. The pupil's work sheet entitled "Shopping for a School Breakfast" dealt with an activity not so apt to be carried out by the mediocre teacher but one which might challenge the superior teacher.

The Try-Out

Before a lesson was considered ready to be broadcast to all the schools in Cleveland, it had to be given a trial broadcast. These so-called try-outs were carried out differently at the various curriculum centers. Some were broadcast from the regular studios of WBOE, with a small group of representative schools receiving the lessons and offering suggestions and criticisms. At other centers the lessons were broadcast over a public address system to one class only, the script writer sitting in the classroom to observe the children's reactions while another person read the manuscript outside the room.

Neither of these methods were considered suitable for the arithmetic lessons. In the first place many of the lessons, such

as the Halloween Party, Planning for Christmas, Community Fund Collection and Seed Order Collection, were planned with a seasonal background. As the trial broadcasts took place about two months ahead of time, it can readily be seen that these lessons were very definitely out-of-season at the time of the try-outs. It was considered inadvisable to expose a number of schools to an artificial situation, if one school could serve the purpose. In the second place Milford School did not have a public address system at the time of the first trial broadcasts. Although not as ideal in setting as the aforementioned methods, the arithmetic try-outs were managed quite well by erecting a screen in the back of a classroom, with the script writer reading from behind the screen. Thus she and her manuscript and stop watch were concealed from the eyes of the children, but the children could be seen very easily by her through the cracks in the screen. In this position the radio teacher could watch the reactions of the pupils to her questions and directions and could time the pauses necessary for pupil and teacher activity.

The classroom teacher who was to help in the try-out was given the teacher's guide sheet and no other directions. The children were told that they were going to help in trying out a lesson which was to be broadcast to all the elementary schools in Cleveland at a later date. They were then told to pretend that it was a certain time of the year, if the lesson was of a seasonal type, and to pretend that the voice they heard was coming from a radio.

Besides the classroom teacher, pupils, and script writer, there were present at these try-outs, the special supervisor of mathematics, the principal of Milford School and two other elementary school principals. The supervisor and three principals were each given a copy of the script so that they could make any notations which they thought advisable as the lesson proceeded. Immediately following the trial "broadcast" there was a conference period in which suggestions for improvement were offered. If many changes were made in the script, the lesson was retried with another class. However, if only minor changes were made, it was not thought necessary to do this. As might be expected the first lessons were often tried two and three times but as the writer became more skilled in the handling of materials, and the techniques and methods particularly adaptable to radio, retrials were fewer and farther apart. For the very first lessons small groups of children were called into the radio teacher's office during the process of writing the manuscript and various questions and directions were put to them. This was an aid in keeping the lessons within the child's interests and understanding.

The Broadcast

As the series of lessons under discussion was presented from the Cleveland Board of Education's privately owned station, there was not the necessity of having each lesson conform to an exact time length. Most of the lessons originating from station WBOE were scheduled a half hour apart and as the average length of the arithmetic lessons was fifteen minutes it did not matter if they "ran over" a few minutes. The time between lessons was usually filled by broadcasting electrical transcriptions. Thus, the problem of the commercial broadcaster, of finishing on the exact fraction of a minute, was eliminated. Naturally this made for a greater ease on the part of the radio teacher. It was thought inadvisable, however, to have the lessons deviate more than five minutes because of the inconvenience that might be caused in individual buildings where departmentalization called for conformity to a definite time schedule. Then, too, as most buildings had but one radio, ample time had to be allowed between lessons for one class to leave the radio room and another class to go in and be seated; or in buildings where the radio was moved, the time necessary to transport the set from

one room to another had to be taken into consideration.

Although the broadcast takes only about fifteen minutes, the hours of work that go into the preparation of a script can be rendered almost worthless by a poor delivery at the microphone. It is stated in the pamphlet "WBOE, Radio Serves Education"

> The most successful broadcasters have been teachers of merit who are able during the broadcast to visualize a classroom of pupils and to image their reactions.—Such a teacher must be able to write the scripts or should have an intimate part in their preparation. Merely hearing a finished script is not an adequate preparation for broadcasting. The qualities of personality and the reactions of the broadcaster need to be incorporated with the script for superior results.
>
> The so-called radio personality seems to be the resultant of a pleasing voice, free from nerousness and faulty breath control, with the type of flexibility and inflection necessary to give variety and keep the voice interesting to the listening ear, combined with a fine timing sense for the raction time of listeners. The good broadcaster is not hurried, is natural, animated, and has a sympathetic understanding of the listener.

The ideal set forth in the above paragraphs is a hard one to attain, but a valuable one for which to strive.

Types of Arithmetic Activities by Radio

A variety of types of activities is necessary to a well rounded program in arithmetic. Among the different types presented over the radio were:

1) Planning an Activity Which Was Later Carried Out
2) Solving the Problems Encountered by Another Group in carrying Out an Activity
3) Problem Analysis—Word Problems
4) Developing the Concept of a Process
5) Developing a Step in a Process
6) Motivation of Drill Lessons
7) Informational Lesson
8) Review Lesson—Question Bee

Evaluation of the Results

The contributions which the radio arithmetic lessons have made to the pupils and teachers should be judged in the light of the objectives of the program. These objectives may be summed up as follows:

1) to assist the teacher in putting into practice the philosophy underlying the arithmetic program
2) to help the teacher in interpreting and carrying out specific objectives of the course of study
3) to vitalize instruction
4) through the above objectives to bring about the improvement of teachers in service

Content, techniques, procedures, devices, and organization and presentation of material, all were evaluated before and during the process of writing the script. Each completed radio lesson was then evaluated during the trial "broadcast" by the Try-Out Committee. When the time came for putting the lessons on the air, it was the practice of the supervisor of mathematics to hear and observe as many lessons as possible in some school in the field. A variety of types of schools was selected for these visits so that the supervisor might gain his impressions from a representative sampling of the system as a whole. The reception of the lessons by the children and the comments of the teachers and principals together with the suggestions of the supervisor were brought back to the script writer. These suggestions and criticisms were clipped to the lessons and filed away so that they might be taken into consideration whenever revision of scripts and teachers' guides for future broadcasting might take place. Wherever advisable the suggestions were used in perfecting other lessons still in the process of being written.

In June, 1939, after two semesters of broadcasting, a questionnaire to secure reactions toward the lessons was sent to all teachers who had received the broadcasts.

The responses to this questionnaire were favorable enough to warrant the continuation of the radio lessons.

Place of Radio in Supervision

Articles on the place of radio in the supervision of instruction are conspicuous

only by their absence. Most of the material written on educational broadcasting is approached from the standpoint of radio as a supplementary device for the enrichment of instruction.

Radio is the speediest of all tools in the dissemination of new ideas to the masses. It is true that these broadcasts could not meet the specific problems or current interests of each classroom with equal success. The topics, techniques, devices, and procedures were chosen and incorporated into the lessons because it was thought that they would benefit most teachers and would appeal to most children at the level for which the work was planned What was lost in not meeting individual needs was more than compensated for by the large number of persons who were reached and the frequency with which they were contacted. It would take a supervisor years to arrange for demonstrations covering the many phases of arithmetic activities in the "model lessons" presented over the air.

The place of radio as a supervisory technique must not be considered as a substitution for the functions of the supervising principal or the arithmetic supervisor in visiting classroom teachers at work and in guiding individual teachers through their particular difficulties.

Its major function probably will be to inaugurate new courses of study or major changes in the philosophy of the program. However, the need of "refresher" courses as part of the in-service training of teachers might well be a function of the radio program, probably in three-year cycle form. Another major purpose of the radio as part of the experience of children in certain subject fields is not so clearly evident in the field of arithmetic; but the fact that teachers overwhelmingly favored the arithmetic lessons indicates that the children were securing experiences that helped them to better appreciate, understand, and perhaps to think in quantitative terms.

In the fall of 1942 it was decided to revise the course-of-study in arithmetic once more. Many changes were made, one of which was to reclassify the objectives of the kindergarten-primary division into fourteen levels of achievement instead of nine. Radio lessons were again used to inaugurate the changes made.

When geometry is not studied and taught as it should be, not only shall we fail to achieve the results at which we aim, but we may even produce results the reverse of those desired. Instead of interested attention, close application and accurate reasoning, we may develop carelessness, inattention, mind-wandering, and finally aversion and disgust. Nothing is more wearisome to the normal mind than the attempt to seize relations between data dimly perceived. The distress consequent upon being continually baffled is equalled only by the "exquisite misery of conscious weakness" which is sure to follow. When geometry is not properly studied and taught it is capable of producing just this weariness and the consequent discouraging sense of weakness. On the other hand, if undertaken in the right way it develops the power of independent effort and affords the just consciousness of strength that is born of achievement. . . .

The peculiar simplicity of the fundamental principles of geometry when rightly presented, the ease with which its data may be grasped, and the certainty of its processes render it a peculiarly valuable instrument for intellectual training. There are few mental images so vivid as those representing geometrical concepts, and no logical processes affording greater definiteness and precision than the deductive proofs of geometrical reasoning. The vividness of geometrical images is greatest of course in good visualizers; but even poor visualizers will find geometrical forms the most vivid of their reproduced visual experiences. Parallel lines, vertical angles, cone, pyramid, sphere,—the objects themselves could hardly be more vivid than our mental representations of them, provided, of course, that the real things which these images represent have been fully and accurately perceived. This full and accurate perception of concrete magnitudes, as we shall see presently, is the basis of all real progress in geometry.*

* Hanus, Paul H. *Geometry in the Grammar School.* D. C. Heath and Company, New York, 1893, p. 4.

It All Began Last Winter

By EDNA R. CHAYT
Baltimore, Maryland

IT ALL began last winter when one of my ninth-grade boys said to me, "Why do we have to study mathematics?" A firm believer in the principle that the best defensive is the offensive, I countered with another question, "Why do you study any subject?" He replied, "Well, history helps you understand what you read in the papers." "So," I told him, "does mathematics. Mathematical ideas are just as much a part of an educated person's background as historical ideas." Well-bred child that he is, he didn't express his disbelief in words, but he didn't have to. I have been refuting the look on his face ever since.

I started collecting from the newspapers I read—the Baltimore *Sunpapers* and the Sunday edition of the *New York Times*—and from books, illustrations of concepts found in junior high school mathematics. The fact that the collection was purely for my own "amazement" and that of my pupils is the explanation for its random nature and lack of scholarly documentation.

Examples of squares and rectangles can be found in all classrooms and homes, but polygons are not so common. Peter W. Rainier, in *Green Fire*, an account of emerald mining in South America, says, "All emeralds are hexagons or fragments of a hexagon. Why? Because the emerald mineral happens to crystallize only in that form." In the incredibly far-off days when gasoline companies were urging motorists to drive on sight-seeing trips, Standard Oil Company published a pamphlet on Delaware. One of the sights in Delaware is a wooden reconstruction of a hexagonal stone lighthouse built in 1725 at Rehoboth Beach. A cartoon in the same pamphlet states, "In Delaware's old octagonal schoolhouse, the boys occupied an outer circle of desks facing the wall while the girls in an inner circle faced the teacher." A little boy in a dunce-cap is shown asking, "What corner shall I stand in, teacher?" One of the least common polygons, the pentagon, has been chosen for the floor plan of the new War Department Building just outside of Washington. The Pentagon Building is so large that it is said to include in its equipment Saint Bernard dogs which range the corridors nightly to succor lost travelers.

In one of the mystery stories to which I am periodically addicted, I found this statement, "The room was large and square, and so high as to be almost cubical."

The following example of *perpendicular* was taken from *Green Fire*, mentioned above, "My right boot was rubbing against the cliff which ascended perpendicularly above me."

Baynard Kendrick's blind detective, Captain Maclain, was able to light his own cigarettes because he always held them at the same known angle to his body. You can believe that or not, but you can be sure that a plane's range is a radius, and the war maps marked off into arcs will help make the idea more vivid to children. A timely example of *diameter* is: "Excavation trucks used by the Army with tire diameters of nine and one-half feet require about 3,500 pounds of rubber."

One of *The Sun's* able editorial writers, Mark S. Watson, uses *perimeter* in this way:

> There has lately been reference to the "armored perimeter" which Japan, from the outset, has been constructing around her stolen empire. Because of that empire's watery character, its perimeter is made up of islands dotted with a succession of airfields which are separated from each other by only 100 or 200 miles. This is easy fighter-plane range, so that each field is a support to its neighbors. A limited defensive air

force, with air-borne troops can be shuttled at great speed to whatever part of that perimeter is threatened.

Mr. Watson has no monopoly on this use of the word. Raymond Daniell, in *The New York Times Review of the Week* of May 30, 1943, writes this non-stop sentence:

A vast perimeter stretching hundreds of miles from the White Sea across the steppes of Russia to the Black Sea, across the Mediterranean and along the Atlantic and Channel coasts around France and the Low Countries to Norway's northernmost tip is what the Axis is trying to defend.

Children sometimes feel that the metric system is a purely academic question. This quotation from an article in *The Evening Sun* of December 18, 1942, may help to bring it closer home. (Senator Kilgore, of West Virginia, was advocating greater standardization of army and navy equipment.)

He pointed out that the army generally has adopted metric bores for its heavier weapons—75 millimeter, 90 mm., 105 mm., and 155 mm., while the navy favors even-inch calibers—3-inch, 5-inch, 6-inch, 8-inch and 16-inch guns. The bore of a 3-inch gun is a fraction of an inch larger than 75 mm.; so is that of a 6-inch weapon compared with a 155 mm. field piece.

Even the most casual perusal of the newspapers will yield many examples of per cents in the advertisements. The news articles also utilize decimal per cents, such as "15.37% increase in March postal revenue noted," and per cents greater than 100%, as in "Park Board seeks approval of higher scale—hikes swim fees 150%," and "Holabird spending up 5,000 per cent—Buys at $150,000,000 rate as compared with usual $3,000,000 a year." However, examples of the three types of percentage problems are more difficult to come by. In desperation, I adapted from news stories the three following problems:

1. In a raid on Germany 30 bombers were lost. This was 3% of the number of planes sent. How many planes were sent?

2. 25 submarines attacked a convoy. After the escort ships fired depth charges they were positive they had destroyed 6 submarines, and listed 4 others as probable hits. What per cent were they sure of? If they destroyed the additional 4, what per cent of the submarine pack was destroyed?

3. A commander cannot afford to lose more than 10% of his planes in one operation. If 300 planes are sent on a bombing mission, what is the greatest number of planes that can be lost?

Then, in one week, I found five additional examples. Gabriel Heatter, in one of his broadcasts, gave the number of planes lost in one operation and said that, since the average loss was approximately 5% of the total force, we could easily calculate the original number of planes. In the same issue of *The Sun*—some time during the week of May 24, 1943—were these two statements:

Ten Goucher College students, representing more than ten per cent of the senior class, will take their oaths for service in the Waves and Spars at a meeting to be held today.

Mr. Blakely said last night's scheduled delivery of 3,000,000 gallons (of gasoline) is equal to forty per cent of a normal, pre-war day's consumption in the city.

The editorial page of *The Sun* for May 31, 1943, yielded these two examples:

Among the 131 cases (of meningococcus meningitis) treated at Sydenham Hospital during the first four months of 1943, only ten have died, a case fatality rate of about eight per cent.

If the increases in the individual income tax under the new bill are considered, this country will be drawing $33,647,000,000 from individual and corporation income and excess-profits taxes and from taxes on employment which are forms of the income tax. This sum represents 87.6% of all Federal tax revenues for the next fiscal year. By contrast our excise and customs revenues will amount to but $4,119,000,000 or 10.6% of the total.

Ratio is represented by these three clippings, all from *The Sunpapers:*

The cost ratio as between air freight and the older methods at sea is, by Mr. Loening's figures, about ten to one. But the cost ratio as between land-air express and railway express is already only about seven to one.

An indicated enemy loss ratio of five-to-one has delayed even if it has not averted whatever major offensive move the Japanese may be preparing against Australia or New Guinea.

(33 RAF planes were lost in the raid on Wuppertal, Germany, on May 30, 1943.)

If the usual ratio of losses to the total attacking force held true, it meant that the bomber fleet was approximately the same size as those which pulverized Dortmund and Duesseldorf with a record 2,000-ton blow earlier in the week.

In my reading, admittedly cursory, I have found *proportion* used only as a synonym for *ratio*, never according to the definition, "A proportion is a statement of equality between two ratios." I therefore adapted one of the sentences in this way:

Before our final victory in Africa, this statement appeared in a newspaper article: "The proportion of Italians captured in Africa is 8 to 1 German." Would you say this is a mathematically correct use of the word *proportion*? Why not?

Two of the most wide-spread concepts are the use of X as an unknown, and the equation. An editorial in *The New York Times* of May 30, 1943, commenting on the isolation of biotin, one element of the Vitamin-B complex, includes this statement, "What began as an algebraic 'X' in food ends as a chemical with a formula, a definite compound that can be made at will in the laboratory." "The human equation" is an expression frequently encountered, but a full statement of an equation is not so usual. I have this one from *The Sun:*

In other words, we are now to have a (theoretically) exact equation between materials and the end-products into which materials are fabricated.

In November, 1941, I found these two statements, the first in the *Evening Sun* and the second in *The New York Times Review of the Week* of November 16, 1941, which, in my opinion, express very well the fundamental idea of the equation as a balance:

In an interview last week Stalin said that he saw the production of war materials as an equation, with the production of Germany and the conquered countries balanced by the production of Great Britain and the United States.

The equation establishing equality between the air forces of Britain and Germany has too many unknowns to be worked out here, but its outline can be seen.

Lancelot Hogben, in *Mathematics for the Million*, at the end of his chapter on "How Algebra Began," gives two number puzzles which ninth-grade children with a little guidance can explain algebraically:

1. Think of a number. Multiply the number above it by the number below it. Add 1. Give me the answer. The number you thought of is — (the square root of the answer).

2. Think of a number. Square it. Subtract 9. Divide the answer by the number which is 3 more than the number you first thought of. Give the answer. What is the number thought of?

A fine example of how mathematical theory is applied in practical situations is found in the following excerpt from an article in *The Evening Sun:*

I wish to describe an interesting example of a discovery by way of mathematical theory. An industrial firm was required to make a scientific instrument necessary for the prosecution of the war. Several models were known to the engineers in the blue-print stage, but it was not known whether any one of the models would be adequate or which one was the best. Considerable expense was incurred in building these models. The test was inconclusive.

As a last resort a mathematician of note who was also an engineer was called in. In two weeks this mathematician had derived a mathematical theory of such models that said in effect, "Such and such models and only such models are possible, and type X is the best." This mathematical solution was complete and trivial in cost.

Sometimes the illustrations are amusing. One of my children brought in this poem, which she had found in *Poems* by Christopher Morley:

$(a+b)^2$
Marriage is the square of a plus b.
In other words
a^2 plus b^2 plus $2ab$
Where $2ab$ (of course)
Are twins.

In *The New York Times Book Reviews*, in a review of one of Faith Baldwin's works, these sentences occurred:

Miss Baldwin could not have been a very orthodox student of mathematics. For her, a straight line is never the shortest distance between two points.

In a discussion of women in war industry, *The Sun* reported this story: A girl who worked with a micrometer was explaining to a friend that the micrometer was accurate in very fine measurements, including thousandths of an inch.

"Thousandths of an inch? My, that's tiny," said the second girl. "How many of them are there in an inch?"

"I don't know," replied the first girl, "but there must be millions of them."

My favorite, perhaps because I am an *Alice in Wonderland* fan from 'way back, is the Mock Turtle's account of his education:

Reeling and Writhing, of course, to begin with, and then the different branches of Arithmetic—Ambition, Distraction, Uglification, and Derision.

In my experience, the illustrations given above add interest to mathematics lessons, whether used as motivations, summaries, or simply as material to "sell" mathematics. I think they may help to convince children that mathematical ideas are part of an educated person's background.

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The Place of Experimentation in Plane Geometry

By HARRY SITOMER
New Utrecht High School, Brooklyn, New York

A SOURCE of serious errors made by high school students in the plane geometry classroom is the confusion between arriving at conclusions based on observations and those based on reasoning. A student will state that an angle is a right angle because it looks like that to him. Or he will conclude that two segments are equal because he estimates their lengths to be equal. Too often he fails to understand the nature of his error. He is an ardent believer in the maxim, "Seeing is believing."

This paper has for one of its objectives the exposition of a possible method for resolving this confusion. We believe that the difficulty has its roots in the philosophy of the teacher who considers reasoning or rationalization as the summit and pinnacle of intellectual glory and looks with disfavor on the lowly and menial method of measurement and experimentation. The author has carefully scrutinized this attitude and concludes that it is unwarranted. Such authorities as Klein and Veblen provide a definite place in mathematics for experimentation while pedagogically we have come to recognize a definite value in the physics-of-space approach.

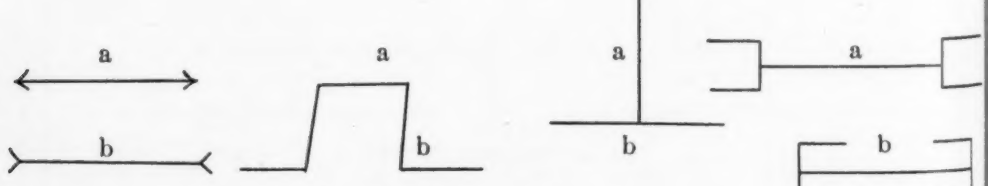
Let us consider two arguments which are often presented against experimentation in the geometry class. The first refers to the existence of visual illusions such as the following:

The unreliability of length judgments by the eye is supposed to cast this type of experimentation into disfavor, but the student's use of a ruler to arrive at the correct conclusion is the very means of demonstrating the error of the judgment.

The second argument casts doubt on the ability of the ruler to judge the equality of two line segments even when refined with such aids as microscopes. The tacit admission that measurement reveals inequalities of the smallest magnitudes is an argument that measurement, if sufficiently precise, can also measure equalities, since the definition of equality between two magnitudes may be that their difference is less, in absolute value, than a predetermined tolerance. Absolute equality is a doubtful concept, incapable of practical demonstration and hence its existence in a real world (which interests our students) cannot be established. It must also be recognized that small inequalities are inherent in the practical interpretations of the axioms and therefore persevere in the derived theorems.

Yet there is a definite value and place for teaching reasoned proofs. That value, however, is not to replace or crowd out proofs by experimentation. To help clarify that value and place the author has adopted this thesis:

1. Proof by reasoning and proof by ex-



Which is longer? a or b?

periment do not conflict with each other. They complement each other. The nature of one's curiosity should determine which of these is to be used.

2. The function of experiment (measurement or observation) is to determine whether a given proposition is true or false. It consists essentially in creating a situation in which the hypothesis was made true and in which the conclusion is tested. Precautions regarding special cases must be taken.
3. A function of reasoning (especially in high schools) applies to propositions which we know or were told are true. We are concerned with seeing how these propositions necessarily follow logically from other propositions. In its best form it is an exercise of the mind which shows how a set of postulates and definitions imply the given proposition. In this sense we have an explanation of the truth of the proposition, a psychologically satisfying experience which derives its satisfaction from seeing the logical unity, or inevitableness of the proposition from previously established propositions.

A second consideration has also led to the pedagogical program for high school about to be described. Experiments and logical demonstrations operate on the same kind of materials: namely, statements of implications. These involve the concepts of hypotheses and conclusions, which are new experiences to the students. Making a diagram from a given statement, identifying the hypothesis and conclusion are a difficult task for the beginner. Reading difficulties complicate these difficulties. The traditional pedagogical approach superimposes on these obstacles still another: that of formulating a proof within the confines of memorized and meaningless (to the student) definitions and postulates.

It is this conglomerate of simultaneously imposed difficulties which causes students to memorize theorems and their proofs in parrot fashion and leaves the unpleasant impression on their formative minds that geometry is a mystery, understood by a few queer and odd creatures like teachers and natural wizards among the students.

To help remedy this situation the teachers of the Mathematics Department of New Utrecht High School have started their year course in Plane Geometry with a three weeks' unit in experimentation. In this unit the student is asked to use rulers and protractors to determine the truth of propositions. The performance of an experiment requires the following steps:

1. Write the statement to be tested in the "If-then" form.
2. Make a diagram in which the conditions of the "if" clause are made true, using rulers, compasses (as dividers) and protractors. Write a statement of what is given.
3. Write a statement of what is to be proved. By means of measurements test the conclusion, which is to be found in the "then" clause.

The form used is the following:

Given

Diagram

To prove

Measurements

1.
2.
etc.

Conclusion:

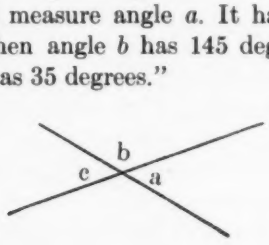
A discussion should then follow as to whether the diagram made was too special a case of the hypothesis. For instance, did the experiment use an equilateral triangle for an isosceles triangle, or a square for a rectangle, or an acute angle for any angle? A comparison of the results of all members of the class then determines whether a conclusion is reliably or tentatively true, or simply false.

The place of definitions is clear in this program. A correct diagram for experimental purposes cannot be made unless definitions of terms are standardized for

all members of the class. To do this a discussion of the requirements of a good definition and the need for undefined terms must be undertaken.

The transition from experiment to reasoning can be made smoothly, gradually and with a sense of accomplishment by choosing a situation and suppressing the measurement. We might choose as our theorem for this purpose "Vertical angles are equal." The experiment is normally made with two measurements. Students are now challenged to prove the proposition with only one measurement. The recitation is generally made as follows:

"Let's measure angle *a*. It has 35 degrees. Then angle *b* has 145 degrees and angle *c* has 35 degrees."



Questions by students elicit the idea that *a* and *b* form a straight angle, while *b* and *c* do also. The argument should then be repeated with varying values of angle *a*.

A second challenge to prove the theorem with *no* measurements stimulates the student to participate in the reasoning processes. The nature of a deduction, and the generality of a reasoned proof become accepted concepts and they arise in a situation familiar to the students. A discussion should follow immediately thereafter relating to the differences and similarities between measurement and reasoning proofs. The conclusions of this discussion are the points in the thesis stated above.

The nature of reasoned proofs unfolds the need for postulates and the student is ready to embark on an adventure within a postulational system.

We may conclude with a statement of the principles guiding us in the choice of theorems for experimental purposes. Best of all are those theorems which are brought forward by the students themselves. When the teacher selects them, they should provide a good scope for definitions and concepts which will be used often in the term. Congruence, similarity, parallelograms, types of angles and triangles and supplements are some of the terms which should be used in experiments. Where possible theorems should be grouped in sets which encourage reasoning voluntarily by the students. For instance, the triangle angle sum theorem might precede the experiment on the complementary acute angles of a right triangle; that each angle of an equilateral triangle has 60 degrees; etc. Finally the set of experiments should include some false propositions, such as "The median of a triangle bisects an angle of the triangle." If one angle of a triangle is twice a second angle, then the side opposite the first angle is twice the side opposite the second," or "A diagonal of a parallelogram bisects a pair of opposite angles."

The experiment serves also to clarify the meaning content of a theorem. Students are sometimes able to give a good proof of a theorem and fail to apply it in a numerical problem. This failure is often due to the lack of understanding of the meaning of the theorem. In such cases an experiment with the theorem will often serve to clarify that meaning. This may be done even after the first three weeks. It has been found useful with such theorems as "The altitude on the hypotenuse of a right triangle is the mean proportional, etc." and "The products of the segments of intersecting chords are equal."

In summary then, a program of experimentation may serve

1. To introduce definitions and concepts at the beginning of the term
2. To form a basis for a gradual transition to reasoning and the concept of a postulational system
3. To establish the truth of a proposition
4. To clarify the meaning of a theorem
5. To teach the complementary relationship between experimentation and reasoning and the functions of each.

A Study of Weights and Measures

By GERTRUDE CUSHING YORKE
Boston, Massachusetts

FOR MOST mathematicians the metric system[1] has a definite lure, because it is completely logical and mathematically perfect. Naturally it appeals strongly to them since it was the brainchild of some celebrated mathematicians who during the French Revolution were commissioned to create a new system of weights and measures worthy of the new order. Unfortunately these mathematicians knew little of and cared still less about the mundane world of trade and industry. They were "idéalogues" as Napoleon styled them, not practical businessmen. In the early eighteen hundreds the French failed to conquer the world, but their "intellectuals" never gave up their dream of world domination in one field, that of weights and measures, and so metric propaganda has worked on unremittingly.

Periodically some educators[2] assume and proclaim that if the use of metric weights and measures were made compulsory in the United States most of the difficulties which children experience with denominate numbers would disappear or at least be largely obviated.

In 1939 the writer undertook a study of this problem to discover what has happened in countries which have made the use of metric weights and measures compulsory by law, imposing penalties for the use of customary[3] weights and measures.

The following three main questions were studied in this investigation:

1. What customary weights and measures are used in trade and industry in the countries studied in continental Europe and South America, where the use of metric units is compulsory?
2. To what extent, in general, are customary weights and measures used in these countries?
3. What further effects does the compulsory use of metric weights and measures have in these countries?

Inevitably certain other highly controversial questions arose in an investigation of this nature. The writer studied the most important of these carefully and formulated opinions in regard to them, but makes no attempt to present authoritative answers. The following questions, however, will be discussed:

4. Why do customary weights and measures continue to be used in the countries investigated in disregard of the laws and in spite of severe penalties?
5. Does the metric substitute plan operate as a system, with most of the well-advertised benefits such as economy in calculations and ease of learning?
6. What modifications or adaptations of metric units occur?

The investigation consisted of three parts. Part I was preliminary background research, including the history of weights and measures and a survey of the literature in the field of the problem. Part II was a study of previous investigations pertinent to the problem, including tabulation, analysis, and interpretation of the data of Halsey's[4] questionnaire study in

[1] The forty-five (not including the franc) weights and measures of the original metric system (1792) and logical extensions of the same.

[2] Worth J. Osburn, *Corrective Arithmetic*, Boston: Houghton Mifflin Company, 1929, Vol. II, p. 75. Lee E. Boyer, "Appreciations in Arithmetic," *Educational Method*, XXI (January, 1942), p. 200.

[3] Those weights and measures which through the centuries have survived on the basis of usage rather than fiat.

[4] Frederick A. Halsey, *The Metric Fallacy*, New York: The America Institute of Weights and Measures, 1919. Halsey received 52 questionnaire replies from these three countries.

Brazil, Argentina and Uruguay, and Kennelly's[5] field study in continental Europe. Part III was an original field study done by the writer in four South American countries; Brazil, Argentina, Uruguay and Paraguay.

The data was carefully tabulated on large master-sheets (18 by 24 inches), each of which carried one of four headings: Weight, Length, Capacity or Surface. These sheets were ruled vertically, so that they had six columns headed as follows: Date, Place, State, Country, Metric, Customary. From these tabulations it was possible to compare the findings numerically, and thus to figure the metric and customary usage on a percentage basis.

A thorough study of metrology showed that while weights and measures always are in a state of change, those which have become especially adapted, whether they are customary or metric, do tend to survive through the centuries.

The conclusions which the writer drew from the field study data may be summarized as follows. The usage of customary weights and measures was approximately 35% of the total usage. The findings further indicated that, in general, the use of metric units in the interior is disregarded to a considerable extent; and evaded, when it is not possible to disregard it, in the cities where government inspection is stricter and the compulsory laws are more rigidly enforced by means of substantial fines or imprisonment, or both. It was found, also, that for the measurement of land, lumber, and fuel-wood customary weights and measures are used frequently. Likewise, for machinery and tools, most of which are imported from the United States and England, the findings showed that customary units are used extensively, as they are, too, for other imports and exports. For river and sea navigation it was found that customary units are in use exclusively, as they are, also, for measuring time. In the measurement of temperature both the Fahrenheit and centigrade scales are used. It was found, too, that children in the schools in three of the countries investigated must know customary as well as metric units, due to the mixed system of weights and measures in use. A comparison of the writer's findings in regard to the three questions studied in this investigation, with the data of Halsey and Kennelly show that, in general, the data of these other two investigators substantiate the writer's conclusions, thus strongly supporting the assumption that these conclusions are valid.

A summary of the general conclusions arrived at by the writer as a result of this investigation are:

1. The customary weights and measures used in trade and industry in the countries studied in continental Europe and South America, where the use of metric units is compulsory, are those which are most convenient, and so have become a part of the people's thinking such as: the inch, the pound, and the foot. Next to these three in frequency of use in South America are the arroba (25.35 pounds)[6] and the palmo (8.66 inches).
2. The extent, in general, to which customary weights and measures are used in the countries studied is indicated by the results of the following investigations. See Table 1.

That is, three different investigators found in South America and in continental Europe customary weights and measures totaling approximately 44%, 35%, and 28% respectively, as per the following table. Obviously, this is not inconsequential because the use of metric units is compulsory in all of the countries studied.

3. The compulsory use of metric weights and measures has had the following effects in the countries studied:
   a. It has created a mixed system.

[5] Arthur E. Kennelly, *Vestiges of Pre-Metric Weights and Measures Persisting in Metric-System Europe 1926–1927*, New York: The Macmillan Company, 1928.

[6] Spanish.

TABLE 1

A Comparison of the Data of Three Different Investigators in South America and Continental Europe, Showing the Extent of Customary and Metric Usage in Countries where the Use of Metric Weights and Measures is Compulsory

Investigator	Place	Per Cent of Customary	Per Cent of Metric
Halsey	South America	44	56
Yorke	South America	35	65
Kennelly	Continental Europe	28	72

The metric units never have been used exclusively, even in France, where the customary tonneau de jauge (100 cubic feet) has been officially adopted,[7] together with customary units of time, angles and navigation, as well as many modified units, such as the demi-kilogramme and the double-mètre,[8] which are permitted to meet the requirements of the law and yet allow the people to think in customary units.

b. It has caused confusion in thinking.

c. It has caused much useless labor in calculating and making conversions.

d. It has caused disregard and evasion of the laws.

e. It has caused a great amount of social and private cost to:

1) The governments, for inspectors and enforcement;

2) Business, industry, trade and commerce, for calculating conversions, and for machinery equipped with both customary and metric gages and devices.

f. Due to the mixed system it has made learning in the field of weights and measures more difficult for children in the schools, necessitating the learning of conversions as well as many additional units.

[7] Kennelly, *op. cit.*, p. 53.

[8] Demi-kilogramme (livre or pound), double-mètre (toise or fathom).

Obviously, questions four, five and six are not within the scope of this study, and the writer makes no attempt to answer them authoritatively, but, because they do arise naturally in a study of this kind, the opinions which the writer has formulated will be presented. In regard to the fourth question, it is the opinion of the writer that customary weights and measures continue to be used in disregard of the laws and in spite of severe penalties in the countries investigated as a result, in general, of three factors. First, the great majority of the people prefer them because they are more convenient than metric units. This is due to the fact that customary weights and measures have developed through the process of natural selection, and thus there is a practical relationship between the units. Furthermore, they are more adaptable to the needs of everyday life and to ordinary industrial and business requirements because they may be multiplied and divided not only decimally, but binarily and duodecimally. Second, it is impossible to enforce the exclusive use of metric weights and measures because the use of customary units by the people is an ingrained habit of thinking and speaking. Fines and imprisonment, besides the confiscation of customary weights, measures and weighing devices have not succeeded in enforcing completely the compulsory metric laws. Third, customary units have been used in the development of many industries, and are used to a great extent in manufacturing, in international trade and in commerce, as well as universally in navigation and sea measurements, and exclusively for time. Therefore the customary units have con-

veniences and values not easily laid aside.

The fifth question, dealing with the metric substitute plan, in the writer's opinion should be answered emphatically in the negative. First, the metric scheme does not operate anywhere as a system. The units most commonly used are those which approximate customary ones such as: the kilogram, the $\frac{1}{2}$ kilo ($\frac{1}{4}$, $\frac{1}{8}$); the meter ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{3}{4}$); the centimeter; the ton; the liter; and the kilometer. The cubic centimeter and gram are useful in the chemical laboratory, but they are not convenient for ordinary purposes. The result is, that only a few isolated metric units,[9] which are halved and quartered as are customary ones, are actually used, supplemented by convenient customary weights and measures, some of which have been legalized in metric values. Furthermore, the metric substitute plan does not make for economy in calculations because, inasmuch as the use of customary units continues, making conversions necessary, there results an increase in calculations (to the extent of conversions) rather than a decrease. Learning, too, in the field of weights and measures becomes more complicated and difficult because, when a mixed system is in use, not only must the metric units be understood and learned, but the customary ones which are commonly used must be learned, together with the necessary conversions.

Modifications or adaptations of metric units, referring to question six, have been made by using binary fractions such as: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$; by using double units and other multiples such as 5 liters; and by adding customary weights and measures, legalized in metric values.

One important implication of this investigation bears upon the educational question of teaching the metric system in the schools of the United States. The evidence beyond any doubt makes clear that it should not be taught as a basic system which will ultimately replace our present one, because:

1. The metric system as such exists only on paper. Actually in use there are a few isolated units, together with their halves and quarters.
2. The twenty-eight metric weights and measures which were legalized in the United States in 1866, are used scarcely at all except in laboratory work, where the gram and cubic centimeter are convenient. For such specialized work these particular units should be taught when needed.
3. The advantages claimed for metric weights and measures are pure academic theory. In practice they have more disadvantages than advantages.
4. Just as a nation's language cannot be taken away and replaced over-night, so its weights and measures cannot be either.

Another implication bears upon the question of whether or not metric usage should be made compulsory, enforced by penalties, in the United States. Again this study gives conclusive evidence that such compulsory legislation would be a great mistake because:

1. The use of twenty-eight metric weights and measures has been permissive for the past seventy-five years, and yet they have made no headway on merit, except in laboratory work.
2. The uniformity of the English system would be lost, and a mixed system would result, causing confusion and useless expense, such as have been experienced in Europe and South America. Furthermore, because the United States is a highly industrialized country, the confusion and expense of a forced change to metric units would be much greater than in South America, which was little developed industrially when the change was attempted there in 1862.

[9] Kilogram (2.20 pounds), Meter (1.09 yard), Centimeter (0.39 inch), Ton (2204.6 pounds), Liter (0.91 dry quart or 1.06 liquid quart), Kilometer (0.62 mile), Cubic centimeter (0.06 cubic inch), Gram (0.035 ounce).

◆ THE ART OF TEACHING ◆

The Present Crisis in the Teaching of Mathematics

By LAURA E. CHRISTMAN
Senn High School, Chicago, Illinois

SAID THE geometry teacher to the boy—"I can't understand why you don't see that. It's as plain as the nose on your face."

Said the boy (who was after all not very stupid about space relations)—"Sure you see it, but I don't."

How can we, the new and old mathematics teachers of the moment, understand why the pupils before us, boys and girls, grown-ups and children, "don't see this or that."

For the war effort, in all its phases, is bringing a tremendous increase in the number of mathematics pupils in private classes and public schools of our nation. Men and women who want to advance into positions of authority and reasponsibility in shops and factories are told to master algebra and geometry or to "brush up on their mathematics." Young people who are interested in aviation and radio are told to understand the mathematics related to their special interests. Adults, young people and children are feeling a strong wind of emphasis on that science which is at once the queen of the sciences and the hand-maiden of the sciences.

As a result many hitherto untrained teachers are being forced into the teaching profession for high school mathematics and "refresher courses" for arithmetic, algebra, and geometry. These new recruits to our mathematics teaching ranks may share with us the opportunity of really starting, in thousands of open minds of our nation, the finest values both immediate and cultural of this training in "the science of necessary conclusions."

What golden rules of mathematical pedogogy shall we older teachers of the country repeat for our own continued guidance and for the help of new teachers coming into our departments at such a time?

With something of this in mind, I handled some of my favorite books in the Dummer library. (The Dummer library has been organized by Mrs. William F. Dummer and placed at the disposal of Chicago teachers for the last twelve years.) Through the books on its shelves I have learned to know something of great mathematics teaching of earlier generations for "all the children of all the people."

I wanted to find out what we, the old and new mathematics teachers of this present day and generation, could best keep in mind under the whirlwind circumstances which surround us, in this time of dangerous opportunity.

First of all I turned to a little paper-covered reprint of Speer's Arithmetics. W. W. Speer was at one time teacher of mathematics in the Cook County Normal School.

On page one I read:

> The products of the senses, especially those of sight, hearing, and touch, form the basis of all the higher thought processes. Hence the importance of developing accurate sense concepts. . . . The purpose of objective thinking is to enable the mind to think without the help of objects.—Thomas M. Balliet.

So thinking begins with accurate sense concepts. Then mathematical thinking must begin with mathematical sense concepts which themselves come from experiences in number and form.

As I turned the pages of Mr. Speers' books I saw these experiences, arithmetical and geometrical, well planned and developed to help the pupil gain accurate sense concepts. And I realized that in many cases even in our more advanced teaching, refresher courses based on imperfect mathematical preparation will call for "refresher" experiences in number and in form devised sometimes on the spur of the moment, but carrying conviction to our pupils' minds and faith in the ultimate reasonableness of what we teach. For mathematics, while an abstract science, is a science abstracted from many experiences for over 3000 years. Here in our war efforts we have at *one and the same time* many such experiences taking place. But in addition to reporting these experiences, how can our mathematics pupils be taught to use their new work experiences to illuminate their understanding of mathematics as well as to use their mathematics to improve their work?

I turned next to the writings of Mary Everest Boole, that great mathematics teacher who spent most of her life "trying to teach mathematics to duffers." In her "Preparation of the Child for Science" the first chapter deals with "The Scientific Mind." Beyond the emphasis which Mr. Speer rightly places on accurately observed and recorded experiences, Mrs. Boole tells us of the process of introducing a concept or idea into a child's mind and the similar process of developing mathematical induction in our pupils' minds.

She says on page 87:

> Between the process of introducing a concept or idea into a child's mind and its bringing forth as an intelligible entity which the conscious intellect is to handle and reason about there should intervene a period of gestation, during which the idea is left dormant and passive in the child's mind; any attempt at premature interference is likely to result in abortion. We are not here reasoning by analogy; we have independent experience of the need of intellectual gestation. I am using physical gestation merely as the readiest way to convey a clear notion what is the doctrine of medical psychology on this subject.

In another place she speaks of the eternal truth of mental pulsation, from experience to abstraction and back again to experience.

She emphasizes repeatedly the idea of a time lapse between the introduction of a concept and its development, the presentation of an experience and its emergence as a conviction of mathematical certainty. Then almost immediately she plunges us into Chapter II, "The Preparation of the Unconscious Mind," and on pages 65 and 66 says:

> Of arithmetic and algebra I shall say little here, as they are treated in another chapter. Only one point I will lay stress on. Many a life of intellectual muddle and intellectual dishonesty begins at the point where some teacher explains the rule for Greatest Common Measure to a child who has not had the proper basis of sub-conscious knowledge laid in actual experiences. Therefore, if you value your child's future clearness in science, trust no teacher to tell him anything about G. C. M. or L. C. M. till you have ascertained that he is able to find, easily and accurately, by means of compasses, the longest length that will repeat exactly into each of two unequal given lengths, and the shortest length into which each of two given unequal lengths, will fit.

But all of this involves a wise reaction by teachers of little experience. How can we know most quickly either that our pupils have not enough experience of form and number or that we have not brought it out well?

If we give a test and the *median* is below our expectations we have either demanded too much of ourselves and a class with the preparation ours has or we have not wisely adjusted our teaching procedures to the vagueness of our pupils' earlier impressions of form and number, to their need for further mathematical experiences, and their further need for time to recall and abstract these experiences.

But can you get anywhere in the requirements of a particular course or job on the basis of individual experience alone? "No," says Mary Everest Boole with clear emphasis. "Science does not claim that pupils shall be told nothing, but shall find out everything for themselves.

What science really does claim in this matter is that a clear line of demarcation shall be preserved between what the individual has observed and what he has learned at second hand. The claim that children shall find out everything and be told nothing is palpably absurd."

However she distinguishes further between scientific development and mathematical development when she writes:

> The general (mathematical) truth is seen in the particular instance, and it is not confirmed by the repetition of instances; . . . the perception of such general truths is not derived from an induction from many instances, but is involved in the clear apprehension of a single instance.
>
> One singular characteristic of mathematics is the automatic power of self-correction which the mind possesses in relation to it. No such thing is possible as the existence of either a persistent or a widely spread error, or a serious difference of opinion, as to the result of a calculation. And this is not due to any special immunity from error which we have in connection with the subjects called mathematical. Every schoolboy knows that it is as possible to make a mistake in a sum as in any other exercise; and the greatest mathematicians occasionally make mistakes in calculations. But there seems to be some mysterious court of arbitration within man which detects mathematical error. Any two persons who have come to different conclusions as to the result of a calculation can find out which of them was mistaken, without appealing to external authority. Nor need they examine any outer facts; they need look at nothing outside of themselves except the slates or papers which contain the register of their own successive mental acts. They judge between themselves, and judge infallibly.

And so from these two wise teachers, W. W. Speer of Cook County Normal School and Mary Everest Boole of England and Ireland, I gleaned these three important axioms of pedagogical truth for mathematics teachers, new and old.

1. Give them mathematical experiences if you can't build on experiences they've already had.

2. Require an understanding of new concepts and abstractions from new experiences in new fields at a proper time interval after the experience has taken place.

3. When forced by lack of time to dictate minimum essentials on authority rather than to develop them by experience, teach the pupil as young as possible to discriminate between first-hand experience of his own and second-hand experience even of his teachers and his parents.

MATHEMATICS then has had four great ages: the Babylonian, the Greek, the Newtonian (to give the period around 1700 a name), and the recent, beginning about 1800 and continuing to the present day. Competent judges have called the last the Golden Age of Mathematics.

Today mathematical invention (discovery, if you prefer) is going forward more vigorously than ever. The only thing, apparently, that can stop its progress is a general collapse of what we have been pleased to call civilization. If that comes, mathematics may go underground for centuries, as it did after the decline of Babylon; but if history repeats itself, as it is said to do, we may count on the spring bursting forth again, fresher and clearer than ever, long after we and all our stupidities shall have been forgotten.*

* From Bell's *Men of Mathematics*, page 18.

The National Council of Teachers of Mathematics

REPORT OF THE TREASURER

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For the Year, February 1, 1942 to February 1, 1943

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Union National Bank of Macomb	$1580.15		
Savings Bank Deposit	821.28		
Commonwealth Edison Bond, 3½% June 1, '68	2073.88		
	$4475.31		
Net loss on Book Value of Bond	1.80		$4473.51
Receipts for the year:			
MATHEMATICS TEACHER, W. D. Reeve	$1395.81		
Bureau of Publications, Yearbooks	2003.38		
Interest on Bond	70.00		
Interest on Savings	14.90		$3484.09
			$7957.60
Expenditures for the year:			
San Francisco Meeting:			
Directors' expenses	$ 138.11		
Program Speaker	50.00		
Local Committee	92.49		
Printing Programs	51.85		
Incidentals	32.14	$364.59	
Denver Meeting		135.16	
New York Regional Meeting		99.50	
President's Office		125.00	
Seventh Yearbook Committee		50.00	
Eighteenth Yearbook Committee		250.00	
Chairman State Representatives, Salary		500.00	
State Representatives Committee		100.00	
Secretary-Treasurer's Office:			
Postage and Supplies	150.00		
Stationery	50.00		
Incidental expense	6.90		
Secretary Service	300.00	506.90	
Contingencies		11.00	$2142.15
			$5815.45

Statement of Assets in Treasurer's Office
February 1, 1943

Commercial Bank Deposit	$1537.69		
Savings Bank Deposit	2205.68		
Commonwealth Edison Bond, 3½% June 1, '68	2072.08	(1942)	(1941)
	$5815.45	$4475.31	$6595.00

(signed) EDWIN W. SCHREIBER, *Treasurer*

The above report has been audited and found correct.

(signed) W. S. SCHLAUCH, *Auditor*

EDITORIAL

Dr. John A. Swenson — Mathematician and Educator

NEWS HAS come to us that Dr. John A. Swenson has retired from active teaching and that he will devote the leisure he has so well earned to the further development of the creative mathematical pedagogy for which he has been noted. The National Council takes this occasion to congratulate Dr. Swenson on a long, successful career and one which has given to mathematical education an endowment of original ideas for the future.

We asked Dr. Edna E. Kramer of the Thomas Jefferson High School in Brooklyn, long associated with Dr. Swenson, to gather some facts about his life that would be of interest to our readers, many of whom are either former students or old friends of Dr. Swenson. We are indebted to her for the following tribute:

A few biographical details concerning this master teacher will indicate the influences which shaped his point of view. In the closing days of the era which knew Riemann, Weierstrass, Poincare and Felix Klein, a dreamy-eyed, tow-headed boy helped his brothers farm the land in southern Sweden. Dr. Swenson's birthplace was Södra Wram, in the province of Skane, separated by the narrow expanse of the Sound from Elsinore, where Shakespeare located his immortal symbol of reasoning, introspective youth.

Young John's mother early recognized his talents and he was sent to the technical high school in Malmö. Scandinavian progressiveness was felt in the schools of the day, and in the mathematical curriculum, texts and methods, the youthful scholar found the germ of "integrated mathematics."

Then, as now, the United States represented the land of opportunity and thus, instead of completing his education in his native land, John, at the age of 17, made his way to these shores. There followed a difficult financial struggle for the boy. In Viking tradition, he found work on ships. No task was too humble. The odd jobs he took were often fraught with adventure. Once he ground the mill for an "Alchemist" whose particular claim was to turn salt into gold. Again, it is amusing to hear him relate the experiences of the summer spent as the substitute for a Lutheran minister.

Soon, however, he found his place as a student at Columbia University, where he was given a number of special scholarships. He sailed, at lightning speed, through the courses for bachelor's and master's degrees. His ability as a mathematician was at once apparent and he was appointed an assistant in mathematics and astronomy. He continued his study of higher mathematics, astronomy and mechanics and completed the requirements for the Ph.D.

At this point he suppressed the normal instinct of the scientist to devote himself to pure discovery and decided to pursue instead the more difficult, less glorious task of secondary education. His first experience in high school instruction was at the Boys' High School in Brooklyn, where the best of the academic mathematical pedagogy of the day was practiced. In 1914 he left this school to become the chairman of the Mathematics Department of Wadleigh High School, New York City. This was a girls' school, and perhaps the sudden contrast in the interests of the new pupils, as compared with those at Boys' High, caused Dr. Swenson to stress the rigors of mathematics less and its beauties more. Whatever the facts, it was at Wadleigh High School, in 1920, that he launched the experiments which finally culminated in the Integrated Mathematics Series. The report of the National Committee on Mathematical Requirements was the basis of the early phase of Dr. Swenson's work found in his "High School Mathematics," published by the Macmillan Company.

As a result of continued experiment, he modified and changed these first techniques to a radical extent, and in his "second period," Edwards Brothers published his "Integrated Mathematics, Books I, II, III, V." Research on the educational implications of this work earned him the doctor's degree at Teachers College, Columbia University. He was appointed an "Associate in Mathematics' at Teachers College and held this position for many years.

More recently he has progressed still further in experiments conducted by him as chairman of the mathematics Department of Andrew

Jackson High School, Queens, New York City, and we may now anticipate publication of Book IV and a revision of the rest of the series.

During these years of professional achievement, Mrs. Swenson, a son, and two daughters, and now four very young grandsons, have made his personal life a most happy one.

We all know the axioms on which John A. Swenson has built his work. Some of them are:

Teach the elements of all mathematics, including the higher branches, at the high school level.

Integrate all parts of mathematics thoroughly —no water-tight compartments.

Emphasize the modern in mathematics, de-emphasize the traditional.

Use analytical methods rather than synthetic.

Teach the fundamentals of differential and integral calculus to every high school pupil capable of understanding it.

As for the conclusions which he has drawn from these axioms, it must seem to his disciples that he has been a Shakespeare of mathematical literature, a Michelangelo of mathematical art. As in the case of these classic greats, it must be impossible for contemporaries to appraise the full magnitude of the master's work. Nevertheless, we must admit that even in a brief score of years, Dr. Swenson's teachings and writings have brought about tremendous changes for the better in the content, method, and spirit of mathematical education in many schools. We may expect those ideas which have taken such firm root in our day to flourish and to influence our thinking throughout the years to come.

There is no doubt that when the history of mathematical education in this generation is written Dr. Swenson's name will have a large place. No greater devotion to the cause of mathematics has ever been among our contemporaries. This is a tribute to the highest quality to which we may all aspire.

W. D. R.

14th Yearbook

of the National Council of Mathematics:

The Training of Mathematics Teachers for Secondary Schools in England and Wales and in the United States

by IVAN STEWART TURNER, Ph.D.

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IN OTHER PERIODICALS

By NATHAN LAZAR
Midwood High School, Brooklyn, New York

Bulletin of the Kansas Association of Teachers of Mathematics
October, 1943, Vol. 18, No. 1.

Preliminary Report of the K.A.T.M. Committee on the Improvement of Instruction in Mathematics.

1. "Three Reports Dealing with Mathematics in the War Effort," pp. 67–69.
2. Price, G. Baley, "Mathematics and the War," p. 70.
3. "Comments and Suggestions from Kansas Teachers," pp. 71–73.
4. "Essential Mathematics for Minimum Army Needs," pp. 73–80 (concluded in item 4 of next issue).

December, 1943, Vol. 18, No. 2.

1. Hassler, J. O., "Teaching Mathematics Effectively for War or Peace," pp. 83–84.
2. Lt. Col. G. Everett Hill, Jr., "Mathematics for the War Effort," pp. 85–87.
3. Eberhardt, Paul, "Mathematics in Air Navigation," pp. 87–88.
4. Preliminary Report of the K.A.T.M. Committee on the Improvement of Instruction in Mathematics (conclusion), pp. 88–89.
5. Mossman, Thirza, "Mathematical Rearmament," pp. 89–90.
6. Hesse, Emma, "A Reorganized Course in Plane Geometry," pp. 90–95.

National Mathematics Magazine
November, 1943, Vol. 18, No. 2.

1. Hume, Alfred, "A Plea for Analytical Mechanics," pp. 50–52.
2. Rutt, N. E., "On Derived Sets," pp. 53–63.
3. Brauer, Alfred, "On Certain Limits," pp. 64–66.
4. Miller, G. A., "A Seventh Lesson in the History of Mathematics," pp. 67–76.
5. Zant, James H., "Mathematics in a World at War—A Challenge to Mathematics Teachers," pp. 77–80.
6. Boyer, Carl B., "Fractional Indices, Exponents and Powers," pp. 81–86.

December, 1943, Vol. 18, No. 3.

1. Reese, R. C., "Quadratic Equations in Engineering Problems," pp. 99–105.
2. Straiton, A. W., "An Application of Fejer Summability," pp. 106–107.
3. Moorman, R. H., "The Influence of Mathematics on the Philosophy of Spinoza," pp. 108–115.
4. Stark, Marion E., "We Look Before and After," pp. 116–119.
5. Rutt, Norman E., "Grades and Distribution," pp. 120–126.
6. Hatch, Hollis D., "Calculus Looks at the Barometer," pp. 127–129.

School Science and Mathematics
January, 1944, Vol. 44, No. 1.

1. Anderson, Theodore W., "A New Approach to Teaching Arithmetic in the Upper Grades," pp. 80–78
2. Nyberg, Joseph A., "Notes from a Mathematics Class Room," (continued), pp. 79–82.

Miscellaneous

1. Asbury, F. C., "Points and Squares," *School* (Secondary Edition), 32: 231–236, November, 1943.
2. Doherty, A. G., and Shaw, L. A., "Applying Music to Number Work," *School* (Elementary Edition), 32: 320–322, December, 1943.
3. Harding, L. W., "Shall We Speed Up the Three R's?", *Progressive Education*, 20: 311–315, November, 1943.
4. Hove, P. W., "Instructions in How to Study Geometry," *School* (Secondary Edition), 32: 329–331, December, 1943.
5. Johnson, J. T., "Evaluation of Research on Gradation in the Field of Arithmetic," *Journal of Educational Research*, 37: 161–173, November, 1943.
6. Mason, M. G., "Arithmetic Problems Suggested by the Sale of Stamps and Bonds," *Baltimore Bulletin of Education*, 21: 27–28, September, 1943.
7. Pritchard, J. W., "Motor Performance as a Chance Factor in Test Scores," *Journal of Educational Research*, 37: 181–192, November, 1943.
8. Schlotzhauer, F. H., "Drill Devices in Arithmetic," *School* (Elementary Edition), 32: 327–329, December, 1943.
9. Thomson, J. R., "Some Alternative Methods in Analytical Geometry," *School* (Secondary Edition), 32: 237–240, November, 1943.
10. Walsh, B., "Daily Tests for Speed and Accuracy: Arithmetic; Grade V," *School* (Elementary Edition), 32: 252–253, November, 1943.
11. Williams, C., "Arithmetic Flies on the Beam," *Ohio Schools*, 21: 350–351, November, 1943.

NEWS NOTES

The Association of Mathematics Teachers of New Jersey held its 78th regular meeting in the Roof Garden of the Hotel Pennsylvania in New York City on Friday, November 12, 1943.

PROGRAM

"Understanding Our World Through Mathematics"

10:30 Business Meeting
10:45 Understanding Our World Through the Mathematics of Geography and Mapping, Mildred A. Fink, Roosevelt Junior High School, Westfield, N. J.
11:10 Understanding Our World Through the Practical Mathematics of the Senior High School, Ferdinand Kertes, Head of the Department of Mathematics, Perth Amboy High School, Perth Amboy, N. J.
11:35–12:00 New Horizons, Dr. E. H. C. Hildebrandt, Northwestern University, formerly State Teachers College, Montclair, N. J.
12:30 Luncheon
1:45 Importance of Mathematical Studies in the Naval College Training Program, V-12, Dr. J. W. Barker, Dean, School of Engineering, Columbia University, New Special Assistant to the Secretary of the Navy
2:30 The Army's Minimum Needs in Arithmetic, Dr. W. A. Brownell, Professor of Mathematics, Duke University, New Consultant in the War Department

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Committee on Afternoon Program—Professor Virgil S. Mallory, Professor Carl N. Shuster.
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Committee on Bulletin—Dr. Howard F. Fehr, J. Dwight Daugherty, Dorothy DeWitt, Madeline Messner, Mary C. Rogers, Mollie R. Bruce.

The Fall Meeting of the Connecticut Valley Section of the Association of Teachers of Mathematics in New England was held at the Hotel Bond in Hartford, Conn. on Saturday Nov. 6, 1943.

PROGRAM

Morning Session

10:45 Mathematics and the Waves, Ensign Margery S. Foster, W.V(S), USNR Naval Reserve Midshipmen's School, WR, Northampton, Mass.
11:45 The V-12 Mathematics Program at Dartmouth, Professor Bancroft H. Brown, Dartmouth College.
1:00 Luncheon

Afternoon Session

2:00 Business Meeting
2:15 Quality Control in Industry, Professor C. H. W. Sedgewick, University of Connecticut
3:15 The Two Reports from Washington, Mr. Rolland R. Smith, Specialist in Mathematics, Public Schools, Springfield, Mass.

OFFICERS OF THE CONNECTICUT VALLEY SECTION

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L. H. Somers, Vice-President, Avon School.
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Ethelyn M. Percival, Treasurer, Westfield High School.
Maytscherl Walsh, Director, Bulkeley High School.
Laura D. Sargent, Director, Ethel Walker School.

The Women's Mathematics Club of Chicago and Vicinity met on Saturday December 4, 1943, for luncheon at the Picadilly Tea Room. After lunch Dr. Louise Lange, of Wilson Junior College spoke on the topic "Under What Conditions are Certain Fundamental Laws of Algebra Void?"

The hostesses for this meeting were Misses Agnes McNeish of Tilden Technical, Bess Dady of Waukegan High, Gladys Gilliland of Lyons Township High, Lagrange and Ellen Gonnelly of South Shore High.—WINIFRED MEAD.

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Hospitality—Lenore Dolejs, Amundsen High School, Chicago, Ill.
Publicity—Winifred Mead, Lane Technical High School, Chicago, Ill.
Membership—Janette Spaulding, Bloom Township High School, Chicago Heights, Ill.

EXHIBIT COMMITTEE MEMBERS

Dr. Ruth M. Ballard, Clara Haertel, Bernice Von Horn, Neva Anderson

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The Men's Mathematics Club of Chicago and the Metropolitan Area held its second meeting on Friday November 19, 1943 in the Central Y.M.C.A. Mr. Joseph E. Dickman, Director of Science and Visual Instruction of The Chicago Public Schools spoke on "Visual Aids in Mathematics" and Mr. Hans Gutekunst of the J. Sterling Morton High School spoke on "Mathematics of Radio in the Army Air Forces."

OFFICERS 1943–1944

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EXHIBIT COMMITTEE

Glenn L. Anderberg, Chairman, Waukegan Township High School
John P. Esposito, Crane High School
Hobert Sistler, Morton High School

The Mathematics Club of Greater Cincinnati is sponsoring a regional meeting of The National Council of Teachers of Mathematics at the Hotel Sinton in Cincinnati on March the eighteenth. The meeting will open with a "Brunch" at eleven o'clock and will feature an exhibit and talks followed by round-table discussion on elementary, secondary and shop mathematics in the postwar world. Members of the Council in the region of Cincinnati, and all teachers and friends of Mathematics are urged to attend.—MRS. CARMILLE HOLLEY RUSH, President of the Mathematics Club of Greater Cincinnati.

SPRING MEETING OF THE NEW JERSEY ASSOCIATION

The Association of Mathematics Teachers of New Jersey will hold its seventy-ninth regular meeting at the Essex House in Newark, New Jersey on Saturday, April 29, 1944. An all-day convention has been arranged and the following program set up:

General Topic

Mathematics for Wartime and for Peacetime

Morning Session
10:30–12:30

Panel Discussion—
Theme—The Mathematics of our Secondary Schools
Presiding—Dr. David R. Davis, President of Association of Mathematics Teachers of New Jersey
Speakers—Dr. Amanda Loughren, Supervisor of Mathematics, Elizabeth, N. J.
J. Dwight Daugherty—Chairman of Mathematics, Eastside High School, Paterson, N. J.
Dr. Carl N. Shuster—Professor of Mathematics, State Teachers College, Trenton
John W. Colliton—Mathematics Instructor, Rutgers University, New Brunswick, N. J.
President's Annual Report
Annual Business Meeting

Luncheon
Time: 1:00 P.M.

Presiding—Dr. David R. Davis
Guest Speaker—Dr. William Betz, Specialist in Mathematics, Rochester, N. Y.
Topic—Whither Mathematics?

BOOK REVIEWS

Introductory Artillery Mathematics and Antiaircraft Mathematics. By Sophia H. Levy. University of California Press, Berkeley and Los Angeles, 1943, 100 pages.

This book is an outgrowth of the author's experience in teaching classes composed of men of the Fourth Antiaircraft Command, 101st Coast Artillery Brigade (AA), preparing for officer candidate schools in the antiaircraft service. The material was first prepared in mimeographed form and was used in classes totalling approximately fifteen hundred men of whom about one thousand were taught by the author herself. The results were so outstandingly good, and the subsequent history of the students so completely justified the selection of material, that publication was urged. The author has brought to this work a strong interest in engineering and other applications of mathematics as well as wide experience in the preparation of teachers of mathematics. As Associate Professor of Mathematics at the University of California she has not only taught mathematics to engineering students for a number of years, but she has, among other activities, directed astronomical research in which expeditious methods of computation are of prime importance.

The solution of the trial shot problem is the chief objective of the course taken by these antiaircraft men. No one contends that this is the sole problem met in the antiaircraft service. But for men whose mathematical background is far from extensive and whose time is necessarily restricted, good teaching demands the selection of some one problem of major importance and concentration on it. Professor Levy has done this. The book provides a discussion and development of the algebra, geometry, and trigonometry needed for the solution of the trial shot and closely allied problems. Unnecessary topics have been most carefully excluded. Material has not only been admirably selected, but it has been logically developed in a clear, concise, and vigorous style. The first chapter covers this ground in twenty-seven pages, so the treatment is brief. At the same time the development gives the reader understanding as well as results. The goal of the author and of the course for which the book was written is never lost sight of. Every topic is introduced because of its particularly usefulness to the student in the work to follow.

"Part I, Algebra, Geometry, Trigonometry, Coordinates in Two Dimensions," is the first chapter. In it, among other things, will be found concise derivations of the laws of sines and cosines for the solution of triangles. It is to be particularly noted that the "ambiguous case" of oblique triangles (given two sides and the angle opposite one of them) is included. This case is of first importance in the trial shot problem. Also, angles are appropriately measured in mils in all illustrative examples, and an excellent discussion of the mil system of measurement is provided. This discussion is superior to that found in trigonometry textbooks. The radian is introduced for a specific purpose, namely to provide the necessary background for an approximation to the lateral deviation in the horizontal plane.

"Part II, Coordinates in Three Dimensions (Terrestrial and Celestial), Determination of True North" gives necessary information in brief but understandable terms. Visualization is assisted by figures of generous size.

"Part III, Logarithmic Scales, Slide Rules, Including the Crichlow Slide Rule, and Solutions of Triangles Using Slide Rules and Using Logarithms" explains the items mentioned in the title and includes appropriate illustrative material. The student can make use of the seven-inch celluloid logarithmic scales provided in the back cover of the book to follow the explanation. These scales form a working slide rule which the student can use to solve problems if a Crichlow Slide Rule is not available. Examples are fully worked out in the text, solutions being given both by logarithms and by the Crichlow Slide Rule.

"Part IV, Motion of a Projectile, The Trial Shot Problem, Lateral Deviations in Slant and Horizontal Planes" speaks briefly of the flight of a projectile, no derivation of the equations of motion being given. The author states that it "has been included in the hope that it may prove interesting to some readers." Since the "trial shot" involves a projectile, it is certainly well to call attention to some of the characteristics of a moving projectile in this way.

"Part V, Solutions of Triangles of the Trial Shot Problem Using Slide Rules and Using Logarithms" provides fully worked examples with a very useful computing form.

"Part VI, Graphical Solutions of the Trial Shot Problem" forms a chapter of considerable interest. It gives geometric constructions by which the problems concerned could be solved.

The seventh chapter is a "Summary of Directions for Solutions of Triangles and the Trial Shot Problem." This draws together the subject matter of the preceding chapters in a detailed set of directions for solving the trial shot problem.

Pages 76 to 98 contain various tables and formulas for reference such as mils to degrees and radians, degrees to mils, four place logarithms of numbers and four place logarithms of trigonometric functions for angles measured in mils.

This book has been very favorably received by military personnel, both by the men who have studied it as a text and by their training officers as well as others. It should be of great value to the high school or college instructor who wishes to introduce significant applications into his courses. It can be read with profit by every mathematics teacher. The University of California Press is to be complimented not only for the pleasing format of the book, but for making available a book which both in plan and execution is a model of its kind.

H. M. Bacon

Essential Mathematics. By W. D. Reeve. The Odyssey Press, New York. 1943. Price $1.32 postpaid.

This is a combined text and workbook. While it was made to meet the deficiencies which the present emergency has shown exist in the knowledge that pupils have of essentials in mathematics, its subject matter is also important for peace time.

The topics treated include the fundamental arithmetical skills, informal geometry, simple algebra, scale drawing, and trigonometry. These topics are presented in an interesting way and are enriched with many valuable applications which show the uses of mathematics in a great many practical fields.

The book may be used as a workbook or it may be used as an ordinary text. Its content covers the suggestions of the U. S. Office of Education report on "Preinduction Courses in Mathematics" as well as the War Department Report on "Essential Mathematics for Minimum Army Needs."

V. S. Mallory

First Year Algebra. By Raleigh Schorling and Rolland R. Smith, with the cooperation of John R. Clark. World Book Company. Younkers-on-Hudson, 1943. xiv+466 pp.

For those who wish to teach algebra in the ninth grade this new book constitutes an adequate quantity of teaching and drill material for a year's work. Written by a group of experienced teachers and well known authors who have spent many years in the classroom, it reflects the change in emphasis upon certain topics and the addition of new material in line with possible needs in our armed forces.

Emphasis is placed upon meanings with sufficient consideration for mastery of important skill operations. This should help those who study the work to acquire a good foundation for further work in algebra, science or certain special courses like aviation, radio, electricity and the like where a knowledge of algebra is basic.

The book is simply and clearly written and attractively printed.

W. D. R.

Liberal Education. By Mark Van Doren. Henry Holt and Co., New York, 1943. Price, $2.50.

This is a book of interest to all who are interested in liberal education whether they be those who are already supposed to have received a liberal education, their sons and daughters who presumably will wish to be recipients of such an education after the war, or any others who believe that the so-called liberal college constitutes the best type of education for a large part of our youth.

It is quite a problem to decide just what particular college may be best fitted to educate its students most fully. Here Mr. Van Doren takes a stand and offers St. John's College as his choice. Whether one agrees with him or not, the book is sure to stimulate one to think further about the important matter of the place and importance of a liberal education.

Mathematics for Victory. By Virgil S. Mallory. Benj. H. Sanborn & Company, Chicago, 1943. vii+430 pp. Price, $1.68.

This new book is a regular textbook which the author prepared to meet the emergency demands for special work in the fundamentals of mathematics. The topics chosen were selected because of their basic value to those who might enter the armed services. However, these same topics are as basic to an understanding of the fundamental ideas of mathematics which are needed by girls who replace men in industrial positions.

The author wrote this book after a study of army needs in elementary mathematics as given in the report of which he was chairman and which appeared in The Mathematics Teacher for October 1943. The book carefully follows this report in providing practical work in mathematics both for the suggested one-year course and the special one-semester course. It will no doubt be used in high schools for those who have had no secondary school mathematics or at best only a general course, or for pupils who have had sequential mathematics, but who need a careful review. The book is well written and is sure to have a value to those teachers who use it. It contains the essential elements necessary for the purposes outlined in the report referred to above, especially in arithmetic, algebra, informal geometry and numerical trigonometry.

W. D. R.

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